

# FACTS AND IDEAS IN MODERN COSMOLOGY

Yu. V. Baryshev<sup>(1,2)</sup> F. Sylos Labini<sup>(3,4)</sup> M. Montuori<sup>(4,5)</sup> L. Pietronero<sup>(4)</sup> <sup>1</sup>

<sup>(1)</sup> Astronomical Institute of St. Petersburg University, St. Petersburg 198904, Russia

<sup>(2)</sup> Scientific-Educational Union "Earth & Universe", St. Petersburg, Russia

<sup>(3)</sup> Dipartimento di Fisica, Università di Bologna, Italy

<sup>(4)</sup> Dipartimento di Fisica, Università di Roma "La Sapienza"

P.le A. Moro 2, I-00185 Roma, Italy.

<sup>(5)</sup> Dipartimento di Fisica, Università di Cosenza, Italy

*Accepted for publication in*  
*Vistas In astronomy* **Vol.38** *Part.4*, 1994

## Abstract

A review of the principles of observational testing of cosmological theories is given with a special emphasis on the distinction between observational facts and theoretical hypotheses. A classification of modern cosmological theories and possible observational tests for these theories is presented. The main rival cosmological models are analyzed from the point of view of observational testing of their initial hypothesis. A comparison of modern observational data with theoretical predictions is presented. In particular we discuss in detail the validity of the two basic assumptions of modern cosmology that are the Cosmological Principle and the Expanding Space Paradigm. It is found that classical paradigms need to be reanalyzed and that it is necessary to develop crucial cosmological tests to discriminate alternative theories.

## 1 INTRODUCTION

Cosmology as a part of physics is an experimental science. For this reason all reasonable relations in cosmology must have an experimental confirmation. The fast growth of observational data in the last two decades now has made possible the comparison between observable quantities and theoretical predictions. The generally accepted basic assumptions for interpretation of observational data in cosmology remain the *Cosmological Principle* and the *Expanding Space Paradigm*. But recently acute discussions about the validity of alternative theories have been resumed in literature (see e.g. Arp et al., 1990; Peebles et al., 1991; Hoyle et al., 1993; 1994a; 1994b). This is not accidental and indicates a new situation in observational cosmology.

The main cosmological theories are based on the assumption of the homogeneity of matter distribution. The reason is the following. The basic hypothesis of a post-Copernican Cosmological theory is that *all the points* of the Universe have to be essentially equivalent: this hypothesis is required in order to avoid any privileged *observer*. This assumption has been implemented by

---

<sup>1</sup>Send offprint request: to F. Sylos Labini

Einstein in the so-called Cosmological Principle (CP): *all the positions* in the Universe have to be essentially equivalent, so that the Universe is homogeneous. This situation implies also the condition of spherical symmetry about every point, so that the Universe is also Isotropic. There is a hidden assumption in the formulation of the CP with regard to the hypothesis that all the points are equivalent. The condition that all the occupied points are statistically equivalent with respect to their environment corresponds in fact to the property of Local Isotropy. It is generally believed that the Universe cannot be isotropic about every point without being also homogeneous (Weinberg, 1972). Actually Local Isotropy does not necessarily imply homogeneity (Sylos Labini, 1994); in fact a topology theorem states that homogeneity is implied by the condition of local isotropy together with *the assumption of the analyticity or regularity* for the distribution of matter. Up to the seventies analyticity was an obvious implicit assumption in any physical problem. Recently however we have learned about intrinsically irregular structures for which analyticity should be considered as a property to be tested with appropriate analysis of experiment (Mandelbrot, 1982; Pietronero & Tosatti, 1986).

The current idea is that in the observable Large-Scale Structure distribution, isotropy and homogeneity do not apply to the Universe in detail but only to a "smeared-out" Universe, averaged over regions of order  $\lambda_0$ . One of the main problems of observational cosmology is therefore the identification of  $\lambda_0$ , but the observational situation appears highly problematic. In fact in the available redshift surveys a clear cut-off towards homogenization has not been identified. On the other hand the Cosmic Microwave Background Radiation (CMBR), that is one of the most important experimental facts in modern cosmology, has a perfect black-body spectrum (Mather et al., 1994) and it is exceptionally isotropic (Strukov et al., 1992; Smoot et al., 1992), so that many theories of galaxy formation in the framework of the Big Bang model have difficulties in considering the small temperature fluctuations of the CMBR as the seeds that give rise to such complex large-scale structures of matter.

Moreover, long term hopes on classical tests such as  $\Theta(z)$ ,  $m(z)$ ,  $N(m)$  are destroyed by recent observations. A lot of observational data are now available for classical tests up to  $z \approx 3$  and  $m \approx 28$  (while Hubble began with  $z \approx 0.001$  and  $m \approx 19$ ; Hubble, 1929) but there is no clear empirical answer to the question about the geometry of the Universe. What is more, such fundamental questions as the nature of the cosmological redshift are still without empirical answer. Indeed the first point in Sandage's list of unsolved problems for the next two decades is the "proof or not that the redshift is a true expansion" (Sandage, 1987) and this concerns the *Expanding Space Paradigm*. This is why a number of cosmological models are discussed now in the framework of the Big Bang as well as of alternative theories (see e.g. Narlikar 1987, 1989, 1993; Harrison, 1993; Hoyle et al., 1993; 1994a; 1994b; Narlikar et al., 1993; Peebles et al., 1991, 1994; Coles & Ellis, 1994).

The present paper reviews cosmological theories and observational tests which could be performed to decide between world models. In *section 2* we describe the empirical basis of cosmology and the theoretical framework of modeling the Universe. *Section 3* is devoted to the analysis of the basic hypothesis of the cosmological models and their predictions for observations. In *section 4* we compare the observational data now available with the predictions of different models. Finally in *section 5* we summarize the situation and formulate the main conclusions.

## 2 GENERAL BASIS OF COSMOLOGY

The observational data and contemporary theoretical physics are the basis of cosmology. Any given cosmological model must be checked by comparison with observations. We divide the cosmological tests into crucial and parametric ones. The first deal with the fundamental basis of any

cosmological theory. The tests of the second kind give an estimate of the parameters of different models. It is important to stress that in real astronomical data several biases and selection effects occur, and hence, the comparison of theoretical prediction with observational evidence is a very difficult task.

## 2.1 Empirical basis for cosmological theories

Well established observations are the basis of any theoretical cosmological model. The experimental results which one can consider as fundamental empirical facts are:

- the local laboratory physics;
- the global cosmological redshift and its linear dependence from distance;
- the space distribution and motion of galaxies;
- the cosmic background radiation, and in particular the microwave radiation;
- the mean chemical composition of matter;
- ages of different kinds of celestial objects.

It is not a trivial statement that the local laboratory physics is a part of the cosmologically distributed matter. Such deep hypotheses, as Mach's principle and Feynman-Wheeler electrodynamics, are based on this fact. The connection between local and global Universe is the most difficult problem in cosmology and it is still without a definite answer.

The cosmological redshift is the second very important fact. The definition of redshift  $z$  is

$$z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}} = \frac{\nu_{emit} - \nu_{obs}}{\nu_{obs}} \quad (2.1)$$

where  $\lambda_{emit}$  (or  $\nu_{emit}$ ) is the wavelength (or frequency) of the light emitted from a light source, and  $\lambda_{obs}$  (or  $\nu_{obs}$ ) is the wavelength (or frequency) of the light observed. This observed redshift  $z$  has many interpretations, but, in any case, any physical interpretation should be kept well distinct from the observational measurement, as stressed by many authors (see e.g. North, 1965; Harrison, 1981, 1993). The observed redshift-distance relation was established by Hubble (1929) for small  $z$  and small distance  $r$  in the form

$$z = \frac{H_0}{c} r = \frac{r}{R_{H_0}} \quad (2.2)$$

where  $H_0$  is the Hubble constant ( $50 - 100 kmsec^{-1} Mpc^{-1}$ ),  $c$  is the velocity of light and  $R_{H_0} = c/H_0$  is the Hubble radius. Modern estimates of the value  $H_0$ <sup>2</sup> will be discussed in *section 4.1.1.*

The distribution and motion of galaxies in space has been investigated very intensively in the last few years. Several recent galaxy redshift surveys such as *CfA1* (Huchra et al., 1983), *CfA2* (De Lapparent et al., 1988; Da Costa et al., 1994; Park et al., 1994), *SSRS1* (Da Costa et al., 1988), *SSRS2* (Da Costa et al., 1994), *Perseus-Pisces* (Giovanelli & Haynes, 1986) and also pencil beams surveys (Broadhurst et al., 1990) and *ESP* (Vettolani et al., 1994), have uncovered remarkable structures such as filaments, sheets, superclusters and voids. These galaxy catalogues now probe scales from  $\sim 100 - 200 h^{-1} Mpc$ , for the wide angle galaxy surveys, up to  $\sim 1000 h^{-1} Mpc$  for the deeper pencil beam surveys, and show that the large-scale structures are the common features of the visible Universe. One of the most important issues raised by these catalogues is that the scale

---

<sup>2</sup> We shall use in this paper the normalized Hubble constant  $h = H_0/100 kmsec^{-1} Mpc^{-1}$ .

of *the largest inhomogeneities* is comparable with *the extent of the surveys*, so that the largest known structures are limited by *the boundaries of the survey* in which they are detected. It is remarkable to note, for example, that Tully et al. (1992), analyzing the combined Abell and ACO cluster catalogues, provide evidence of structures on a scale of  $\sim 450h^{-1}Mpc$ , lying in the plane of the Local Supercluster (see *section 4.2.3.*). Hence from these data emerges a new picture in which the scale of homogeneity seems to shift to a very large value, not yet identified.

The best known background radiation is the Cosmic Microwave Background Radiation (CMBR). After Relict and COBE experiments, the CMBR anisotropies and spectrum are well known (Mather et al., 1990; 1994; Melchiorri & Melchiorri, 1994; Gush et al., 1990; Strukov et al., 1992; Smoot et al., 1992). The perfect thermal Planck spectrum of the CMBR and its very small anisotropies are among the most important observational cosmological facts (*section 4.2.4.*).

The relative abundance of chemical elements in the observable Universe represents the most stringent proof on the processes of generating and destroying atomic nuclei. We discuss the experimental data in *section 4.1.5.4.* Another important experimental evidence concerns the age of stellar systems: the age estimates of celestial objects are based on stellar evolution, stellar dynamics and radioactive decay theories. Hence the ages are not purely observational, but rather an indirect evidence. In addition one should not forget that the age of the system can be much more than the age of an element, if the system is open and there is the possibility of replacing dead elements with new ones. A discussion of the modern data will be done in *section 4.1.5.1.*

Now we will consider possible theoretical schemes which could be proposed to explain these data.

## 2.2 A classification of cosmological theories

We shall classify cosmological models according to their answers to the following questions (see Fig.1):

- What is gravity?
- How is matter distributed in space?
- What is the nature of the redshift?
- What is the nature of the Cosmic Microwave Background Radiation (CMBR)?
- What is evolution and the arrow of time ?

### 2.2.1 Gravitation theories

The heart of any cosmological theory is the gravitation theory. Gravity is the only important force beyond galaxy scale. The gravitation theory can be constructed in different ways. Presently there are at least three main approaches to relativistic gravity theories. The first one considers gravity as a property of space-time, i.e. it is the geometry of curved space-time. The second one treats gravity as a kind of matter within the space-time: it is the relativistic field theory in flat space-time. The third approach is based on the direct interaction between gravitating particles. It is important to note that up to now relativistic gravity has been tested experimentally only in weak field approximation. Such well known relativistic effects as the bending of light, the gravitational frequency shift, the pericenter advance, the delay of light signals, rotational effects and gravitational radiation from binary systems may be derived in many reasonable gravitational theories.

Figure 1: A classification of cosmological models in accordance with basic initial assumptions. We have discussed four different cosmological models: Standard Friedmann model, Steady State model, Fractal model and Tired Light model. The basic hypothesis of these models are: the interpretation of gravity, the matter distribution, the nature of cosmic microwave background radiation, the nature of cosmological redshift, the interpretation of evolution and the arrow of time.

According to the geometrical description, gravity is curvature of space-time and test particles move along geodesic lines in the curved space. The metric  $g^{ik}$  of space-time (hereafter simply "space") is determined by the distribution of matter via Einstein-Hilbert equations (see e.g. Weinberg, 1972; Misner, Thorn & Wheeler, 1973). In the framework of geometric theories there are several generalizations of Einstein's equations, such as the famous  $\Lambda$ -term, (see Weinberg, 1989), the additional scalar field in Brans - Dicke's theory (Brans & Dicke, 1961), the  $C$ -field in Hoyle's theory (Hoyle, 1948, 1991, 1993b), the variable mass Narlikar-Hoyle theory, the non symmetrical affine connection, etc. An interesting class of quasi-geometrical theories is represented by the bimetric-theories, in which two metrics coexist; one is the Minkowski metric  $\eta^{ik}$  and the other is the metric of a Riemannian (effective) space  $g^{ik}$  (see e.g. Logunov & Mestvirishvili, 1989). Cosmological solutions of these theories could differ significantly from classic Friedmann models despite the fact that there is an agreement in the weak field approximation. It should be stressed that although the geometry has great success in gravity physics, there are several conceptual difficulties within the geometrical description of gravity, such as: what is the curved measuring rod in curved space ? How do conservation laws work without flat space-time symmetry ? What is the graviton, i.e. the quantum of the gravitational field? Really, we know that the photon is the massless particle of electromagnetic field which carries energy-momentum within the space, but if gravity is the geometry of space, which particle carries energy-momentum through the space? Another problem is the pseudotensorial character of the gravitation energy-momentum.

The tensor field description of gravity by means of symmetric tensor field  $\psi^{ik}$  in flat Minkowski space-time, could resolve these problems, because gravity may be thought as a material field in the space-time. In such an approach, due to the symmetry of Minkowski space, all conservation laws exist; graviton is a massless material particle in space, energy density of gravitational field is positive for free and static fields. The main difficulty, in this approach, is that Lagrangian formalism does not determine uniquely the energy momentum tensor (EMT) of any field. But in the case of gravitation, the EMT is the source of the field (nonlinear character of gravity) and must determine the right side of the field equations. The tensor field theory of gravitation (TFT) has been discussed by many authors (see e.g. Thirring, 1961; Kalman, 1961; Weinberg, 1965; Ogievetsky & Polubarinov, 1965; Whitrow & Morduch, 1965; Deser, 1970; Bowler, 1976; Cavalleri & Spinelli, 1980; Sokolov & Baryshev, 1980; Baryshev, 1988; Sokolov, 1992; Baryshev, 1994b), but it is poorly known among astrophysicists. Within the context of the tensor field theory there are many possibilities for choosing the field and interaction Lagrangians and one can get Einstein-like field equations (Ogievetskij & Polubarinov, 1965) as another scalar-tensor possibility. A detailed review of the TFT and its astrophysical applications will be given by Baryshev (1995a).

The third possibility is the relativistic direct-interaction theory of gravitation, which may be considered by analogy with classical electrodynamics in terms of direct inter-particle action (Wheeler & Feynman, 1949). Within the scope of this theory, there is no such concept as the field and no problem of an infinity in the energy of the electromagnetic field of the point charge. This approach has been considered in details in the textbook by Hoyle & Narlikar (1974) and in the thesis of Pantyushin (1972). The direct interaction theory is particularly attractive for cosmology. In fact, one can interpret the independent degrees of freedom of the field, in the field theory, as the interaction of local systems with the whole Universe.

## 2.2.2 Matter distribution

In this section we discuss the distribution of visible matter (galaxies) as an experimental fact and only later we consider its theoretical implication. It is important to specify this perspective because this field is often strongly influenced by theoretical expectations or "principles". The Cosmological Principle was introduced to avoid schemes in which we would be in a special point of

the universe. This is indeed a reasonable requirement that, however, was interpreted in practice as corresponding to the too strict requirements of isotropy and homogeneity. The present discussion will show that it is risky to adopt principles that have not been tested experimentally. However we are going to see in the end that the essence of the principle is reasonable and correct. What is not correct is the usual mathematical interpretation that was due to the lack of other more subtle concepts that have been developed only recently.

With the Cosmological Principle in mind people began to look at the observational data about galaxy positions holding the idea that, obviously, the distribution of galaxies must become homogeneous above some relatively short length scale. The first observations, like the Lick catalogue, referred only to the angular positions of galaxies in the sky and indeed showed a well defined tendency towards a smooth distribution at relatively large angular scales. The situation appeared to be in agreement with expectations and these data were analyzed via Limber equation to go from angular to space coordinates. Limber equations are based on the assumption that the scale of eventual correlations is much smaller than the size of the catalogue and allow us to reconstruct the real three dimensional properties from the angular ones. In this way a correlation length of  $5\text{ Mpc}$  was identified as the characteristic length for homogenization.

This well behaved situation began to shake with the extensive redshift measurements. The direct test of the actual positions of galaxies in three dimensional space showed in fact the presence of structures like clusters and superclusters, as well as voids, whose sizes were much larger than the previously derived characteristic length. At first the data were questioned as incomplete or referring to rare fluctuations. Contrary to these expectations, however, the improvements of the data made these structures sharper and showed that they are actually everywhere. The first three dimensional catalogues, like *CfA1*, are much more irregular than the angular catalogues. This appeared to be a critical situation, but then, a statistical analysis, based on the Limber type hypothesis, was performed on these three dimensional catalogues and confirmed the existence of a correlation length of  $\sim 5\text{ Mpc}$  above which the distribution is expected to approach to the homogenization. This was, in our opinion, the crucial point when the field entered into a situation of ambiguity and confusion that persists still today. As we are going to see, the question is simple but subtle and requires new concepts that were not available at that time (see *section 3.4.*).

It may be interesting to note that the confrontation with intrinsic irregularity and non-analyticity has been quite dramatic also in other fields of physics like, for example, critical phenomena that required conceptual revolution of the renormalization group to be understood. The additional problem with cosmology, that explains in part the resistance to these new ideas, is that intrinsic irregularities and non-analyticity appear to be in contrast with the Cosmological Principle: we are going to see that this is actually not the case and that, in many ways, the new picture, that will arise from our discussion, is much simpler than the standard one.

### 2.2.3 The nature of the redshift

As we have emphasized above, the cosmological redshift is an observational fact which can be interpreted in different ways. Harrison (1981, 1993) has shown a clear distinction between different possibilities to interpret the observational redshift-distance relation. There are at least four possible mechanisms for cosmological redshift: space expansion, Doppler effect, gravitational effect, and tired light effect.

The space expansion redshift is due to increasing of space volume ("space creation") in an expanding universe. The expansion redshift is determined by the Lemaitre expansion-redshift law:

$$\nu_{obs} = L\nu_{emit} \quad (2.3)$$

$$L = \frac{R(t)}{R_0(t_0)} \quad (2.4)$$

$$(1+z)_{\text{expansion}} = \frac{R_0(t_0)}{R(t)} \quad (2.5)$$

where  $\nu_{\text{emit}}$  is the emitted frequency and  $\nu_{\text{obs}}$  is the observed one,  $L$  is the expansion Lemaitre factor,  $R$  is the value of the scaling factor at the time of emission and  $R_0$  is the value at the time of reception. The ratio  $R_0/R$  means how much the universe has expanded during the time of photon traveling. The redshift (Eq.2.5) could be understood as a result of the effective recession velocity, but not as a result of the ordinary physical velocity in ordinary physical space. In particular the effective recession velocity can be much greater than velocity of light (see e.g. Murdoch, 1977; Harrison, 1981, 1993).

The expanding space paradigm, even if wildly accepted, has been criticized by several authors (see e.g. Milne, 1934), who emphasized that space itself has no existence and probably it is more physically correct to use static space, as in ordinary physics, and consider the expansion of matter as motion in this space. In this perspective the observed cosmological redshift could be due to the Doppler effect, gravitation and tired light effects.

It was shown by Bondi (1947), that in the case of spherically symmetrical distribution of dust-like matter, the space expansion redshift can be expressed as a sum of two parts. The first one is the Doppler effect, due to the relative velocity of the source and the observer. The second part is due to gravitation effect of the total mass inside the spherical ball, with the light source at the center and the observer at the ball surface. So for small masses ( $GM/r \ll c^2$ ) and small velocities ( $v \ll c$ ), the cosmological redshift is given by:

$$z_{\text{cos}} \approx \frac{v}{c} + \frac{1}{2} \left( \frac{v}{c} \right)^2 + \frac{\delta\Phi(l)}{c^2} \quad (2.6)$$

where

$$\delta\Phi(l) = \Phi(l) - \Phi(0) = \int_0^l \frac{GM(l')}{l'^2} dl' \quad (2.7)$$

where  $\delta\Phi(l)$  is the Newtonian potential difference between the observer at the surface and the source at the center of a finite ball with proper radius  $l$ , and  $\Phi(l)$  is the Newtonian potential ( $\Phi(l) = -GM(l)/l$ ). In the special case of the critical homogeneous matter distribution,  $\rho = \text{const.}$ ,  $v^2 = 2Gm/l$  from Eq.2.6 and 2.7, we get:

$$z_{\text{cos}} = z_{\text{Dopl}} + z_{\text{grav}} \quad (2.8)$$

where

$$z_{\text{Dopl}} = \frac{v}{c} + \frac{1}{2} \left( \frac{v}{c} \right)^2 \quad (2.9)$$

and

$$z_{\text{grav}} = \left( \frac{2\pi G\rho}{3c^2} \right) l^2 \quad (2.10)$$

It means that the cosmological redshift does not depend only on the conditions at the source and at the observer, but also on the distribution of matter in the intervening space around the source. Note that the gravitational part of the cosmological shift is redshift and not blueshift, as it was supposed by Zel'dovich & Novikov (1977). It is important that for the calculation of  $z_{\text{grav}}$  in Eq.2.10, we must consider the mass distribution around the source, but not around the observer, and use proper distance to calculate the mass effect. It also strictly follows from the general Mattig's relation between proper distance and redshift (see *section 3.2.4.1*).



To generalize Eq.2.10 in the case of the fractal matter distribution, we can use the equivalence of the points of the structure and the statistically average spherical symmetry around each point. As in the homogeneous case, for calculation of the gravitational redshift, we must consider mass around the source. Hence, for simple fractal structure with fractal dimension  $D$ , the mass in a ball of radius  $l$  scales as:

$$M(l) \sim l^D \quad (2.11)$$

An important consequence of the fractal distribution of matter is the possibility of linear gravitational redshift-distance relation for  $D = 2$  (Baryshev, 1981, 1994a). Indeed from Eq. 2.10, for  $D = 2$ , we get:

$$z_{grav} = \left( \frac{2\pi G \rho_0 R_0 l}{c^2} \right) = \frac{H_{grav} \cdot l}{c} \quad (2.12)$$

where

$$H_{grav} = \frac{2\pi G \rho_0 R_0}{c} \quad (2.13)$$

and  $\rho_0$ ,  $R_0$  is the lower cut-off of the fractal structure. To give an order of magnitude for  $H_{grav}$ , one can choose values for  $\rho_0$  and  $R_0$  corresponding to the typical galaxy parameters, then:

$$H_{grav} = 68.6 \left( \frac{\rho_0}{5.2 \cdot 10^{-24} \text{gr cm}^{-3}} \right) \left( \frac{R_0}{10 \text{kpc}} \right) \text{km} \cdot \text{sec}^{-1} \cdot \text{Mpc}^{-1} \quad (2.14)$$

In the case of a static spherically symmetric mass distribution of radius  $r$  and mass  $M$ , for the gravitational redshift of the light by the source at the surface and observed at infinity is given by:

$$\nu_{obs} = E \nu_{emit} \quad (2.15)$$

$$E = \left( 1 - \frac{2GM}{rc^2} \right)^{1/2} \quad (2.16)$$

$$(1+z)_{Gravity} = \left( 1 - \frac{2GM}{rc^2} \right)^{-1/2} \quad (2.17)$$

$E$  is the Einstein gravitation factor, and  $z_g \rightarrow \infty$  when  $r \rightarrow R_g$ .

The exact formula for the Doppler effect caused by the relative motion of bodies in space is:

$$\nu_{obs} = D \nu_{emit} \quad (2.18)$$

$$D = \gamma^{-1} (1 - \vec{\beta} \cdot \vec{n})^{-1} \quad (2.19)$$

where  $D$  is the Doppler factor,  $\gamma = (1 - \beta^2)^{-1/2}$  is the Lorentz factor,  $\vec{\beta} = \vec{v}/c$  is the velocity vector of the moving body and  $\vec{n}$  is the unit vector to observer direction. For purely radial receding motion the angle  $\Theta$  between  $\vec{n}$  and  $\vec{\beta}$  equals  $180^\circ$ , then according to Eq.2.18 we have redshift

$$(1+z)_{Doppler} = \left( \frac{c+v}{c-v} \right)^{1/2} \quad (2.20)$$

Note that Doppler redshift  $z \rightarrow \infty$  for  $v \rightarrow c$ .

The tired light redshift is a result of the photon energy loss due to some unknown physical process. This idea was first suggested by Zwicky (1929) and later discussed by many authors (see e.g. Hubble & Tolman 1935, Geller & Peebles, 1972; Jaakkola et al., 1979; LaViolette, 1986; Vigier, 1988). According to the tired light mechanism, photon energy is depleted in a linear fashion. Hence for a finite distance  $r$  we get:

$$h\nu_{obs} = Zh\nu_{emit} \quad (2.21)$$

$$Z = e^{-\alpha r} \quad (2.22)$$

$$(1+z)_{Tired} = e^{\alpha r} = Z^{-1} \quad (2.23)$$

Where  $Z$  is the Zwicky factor,  $\alpha = H_0/c$  is the energy attenuation coefficient,  $H_0$  is the Hubble constant. Of course, at the present time, there is not any experimental evidence for this kind of energy dissipation, but in principle it could be checked in future experiments.

### 2.2.4 CMBR nature

The three main characteristics of the CMBR are the nearly perfect blackbody spectrum, its exceptional isotropy and that it has an energy content comparable with that of local radiations, such as galactic dust, zodiacal light emission and starlight background, despite the fact that it is an extragalactic radiation. There are basically two different approaches to explain the nature of the CMBR. The first refers to the hypothesis that CMBR is originated in the hot early Universe. In fact in the standard picture of Hot Big Bang theory (HBB), the Universe had expanded from a very dense and hot state. Consequently the space was filled with blackbody radiation. As the Universe expands, the radiation preserves a blackbody spectrum with a temperature that decreases with redshift:

$$T_{obs} = T_s/(1+z) \quad (2.24)$$

where  $T_{obs}$  is the observed temperature,  $T_s$  is the temperature of the epoch with redshift  $z$ . The present temperature of the CMBR cannot be deduced without free parameters: for  $z \approx 10^3$  and  $T_s \approx 3 \cdot 10^3 K$  we get the CMBR with the temperature  $T_{obs} \approx 3K$ . Thus it is pre-stellar radiation which has been cooled by the expansion of space.

According to the second approach the CMBR is the result of an integration of the contributions from an appropriate set of celestial objects. This opposite point of view on CMBR was discussed in alternative cosmological models. The most interesting one is the post-stellar thermalized radiation (see e.g. Hoyle, 1991). If the observed abundance of  ${}^4He$ , that is  $\approx 7.5 \cdot 10^{-32} gram \cdot cm^{-3}$ , has been synthesized from hydrogen in stellar interiors, with an energy release of  $6 \cdot 10^{18} ergs$  per gram of  ${}^4He$  produced, then it gives a cosmic electromagnetic energy density of  $4.5 \cdot 10^{-13} erg \cdot cm^{-3}$ . If this electromagnetic energy has been thermalized, the resulting temperature  $T$  would be about  $3K$ . It is obvious that the alternative redshift mechanisms could lead to cooling process of any radiation in the Universe. Eq.2.24 is valid for Doppler, gravitational and expansion redshifts. In addition, there must be some thermalising processes in the Universe to convert radiation to its thermodynamic state. The mechanism of thermalization is the harder problem in this scenario. In fact dust emission differs substantially from that of a pure blackbody and one is forced to introduce an ad hoc mechanism of thermalization. The simplest way, that is the presence of a huge dust density to make the Universe opaque, is forbidden by the observed transparency up to  $z \sim 4 - 5$ . Hence the blackbody spectrum can be recovered only if an appropriate mechanism of thermalization is assumed. Very long grains must be hypothesized in order to get the needed emissivity but their existence in the extragalactic environment is doubtful. For a discussion of the experimental data on the CMBR see *section 4.2.4*

### 2.2.5 Evolution and the arrow of time

The best known basis for an evolutionary approach to cosmology is the second law of thermodynamics. Irreversible events determine the notion of the thermodynamic arrow of time and the thermodynamic evolution of matter in the Universe. Another type of irreversible event has to be found in the generation of electromagnetic and gravitational radiation. These radiations carry energy from accelerated sources outwards. The time-reversed version of these events, i.e. the

spherically symmetric convergence to a common point, has never been observed. The absence of incoming radiation determines the electromagnetic and gravitational arrow of time in the sense of the absorber theory of Wheeler and Feynman. A discussion of the "correct response" of the Universe has been done by Narlikar (1977) in connection with the definition of the three arrows of time: thermodynamic, electromagnetic and space cosmological expansion. He argued the expansion of the Universe as a whole to prevent thermodynamic equilibrium being reached.

The problem of evolution is also connected with the origin of the chemical elements and with possible variation with time of fundamental physical constants. The origin of the different types of astrophysical objects, such as stars, standard galaxies, extremely active galaxies and quasars, clusters of galaxies, large-scale structures of the Universe, is a matter of discussion for any cosmological model, and one needs to construct evolution scenarios for the life of these objects.

Note also that thermodynamic evolution of the self-gravitating open systems could be a cause of the origin of an order from chaos. This subject has been very poorly developed up to now, while in the real Universe this is the main type of astrophysical object.

### 2.2.6 Main rival cosmological theories

In Fig.1 a classification of cosmological theories is shown according to their answers to the above mentioned questions.

The Big Bang model, based on the geometrical gravitation theory (General Relativity), has thermodynamic and cosmological evolutions due to the expansion of space. The matter distribution is homogenous, the redshift is due to space expansion and the CMBR is the relict of a hot beginning.

The Steady-State model uses geometrical scale invariant gravitation theory or, in some variant, direct interaction theory in curved space-time. The model has three self-consistent arrows of time: thermodynamic, electromagnetic and cosmological. The Universe is globally in a steady state, due to the creation of space with matter. The matter distribution is homogenous and the redshift is caused by the expansion of space. The CMBR can be a relic of newly created particles or a post-stars thermalized radiation.

Fractal cosmology is now at the beginning of its developments. There are some attempts based on the General Relativity framework and many possibilities are still under consideration: it is possible to include this cosmology in the Big Bang theory even if one has to consider that the homogeneity of the luminous matter distribution is not reached up to some hundreds of Mpc. For example, if the dark matter turns out to be predominant and distributed homogeneously at smaller scales, there may be basically no problems with the Friedmann metric, the Big-Bang model, etc., but in this case it should be difficult to explain the forming of very large-scale structures with power law correlations within the Hubble time. In the opposite case, the problem becomes very hard and it has to be reconsidered from the very beginning. Some preliminary ideas will be presented in *section 3.4*. Even if no definite theory is now available, it is interesting to discuss a new approach based on tensor field relativistic gravitation in Minkowski space-time. In any case some fundamental aspects of the current theories of galaxy formation, such as the biased galaxy formation (Kaiser, 1984) and related theories, have fundamental problems with the new picture that now emerges from the data. The main point that we stress in *section 3.4* is that one cannot discuss the properties of a self-similar distribution in terms of amplitudes of correlation. The only meaningful physical quantity is the exponent that characterizes the power law behavior, while the amplitude is an essential part not only of the data analysis, but also of the theoretical models.

The tired light cosmology is based on an alternative redshift phenomenon, due to the still unknown physical process of photon energy depletion in space. The matter distribution is homogenous, the CMBR is post-stellar. Space expansion is excluded, but local evolution is possible.

Unfortunately in the tired light cosmology, the gravitation theory is not, generally, discussed.

## 2.3 Classification of cosmological tests

As we have emphasized above, the main aim of observational cosmology is to compare predictions of cosmological models with available observational data. There are two kinds of cosmological tests according to the two parts of cosmological theories, namely, the initial hypotheses and predicted relations between observable quantities (e.g. redshift  $z$ , flux density  $F_\nu$  or integral flux  $F$ , magnitudes  $m_i$  in filter band  $i$ , angular size  $\Theta$ , surface brightness  $J_i$ , galaxy number counts  $N_i$ ). In this connection we introduce the nomenclature of crucial and parametric cosmological tests. Crucial tests allow us to judge the validity of the basic assumptions of the theories, and parametric ones give experimental estimations of the model parameters. Well known classical cosmological tests, such as angular size-redshift  $\Theta(z)$ , visual magnitudes-redshift  $m(z)$  and number counts of galaxies  $N(m)$ , are parametric ones.

According to our classification of cosmological models (Fig.1), we could consider as crucial the following tests: experimental testing of the relativistic gravity, observations of matter distribution, testing of the nature of the redshift, testing of the nature of the CMBR, direct measurements of an evolution effect. Note that even parametric tests, considered as a unified system, may have properties of crucial tests if there are no parameters of the models which satisfy this system of tests. Of course crucial tests are the most interesting experiments in observational cosmology, but obviously, they are limited by observational technical limits and are not easy to perform. Among these tests the distribution of luminous matter in the three dimensional space has had a strong impulse in this last decade and it will be rapidly developed in the following years. From these data one should have the opportunity to test, at very large scale, the basic assumption of homogeneity of the matter distribution. As we discuss in *section 4.2.2*. these observations can have important consequences also for the nature of the redshift. In modern cosmology there are a lot of parametric tests which deal with different kinds of astrophysical objects in a wide range of the electromagnetic spectrum from radio to gamma rays. In this paper we shall consider only classical tests, because they have furnished the most reliable data (see *section 4*).

## 2.4 Biases and selection effects in astronomical data

For testing a cosmological model, we need to choose astrophysical objects and fix the values of their parameters, which will be used as standards in the test. The main difficulties in this approach are the possible evolution effects of the objects, i.e. the time variation of parameters, and different kinds of biases and selection effects which are present in all astronomical data. Astrophysical biases or selection effects can be divided into two groups: physical effects and technical effects. In their turn, physical effects can be intrinsic and intervening.

The technical selection effects are caused by the technical limits of astronomical devices: telescopes, light receivers, processing electronics. The main ones are: finite angular, spectral and time resolution, finite flux sensitivity, fixed aperture and spectral band, and receiver's noise. A very important selection effect is the so-called Malmquist bias, that arises in flux limited samples and states that the average absolute luminosity of the nearby members of the sample is fainter than that of more distant members (see e.g. Teerikorpi, 1987; 1993; Sandage, 1988).

The next selection effect is the K-effect, which results from the cosmological redshift of the spectrum of distant objects. The K-correction is the magnitude difference between a redshifted and non-redshifted spectral energy distribution, when observed through a fixed spectral interval. The selection is due to the combination of the wavelength shift and the fixed detector effective wavelength. The K-correction for galaxies of different morphological types is necessary to interpret

magnitude-redshift relation and the luminosity function of galaxies (see e.g. Pence, 1976; Sandage, 1988; Yoshii & Takahara 1988; Guideroni & Rocca-Volmerange, 1990).

The background radiation, including extragalactic, galactic, solar system space and earth atmospheric radiation (for ground observations) is a physical intervening selection effect too. Really physical and technical effects work together and can produce very "strange" behavior of directly observed quantities.

An important bias in the detection of galaxies, is related to their surface brightness. Very distant galaxies, that have compact images, will appear just like a star on the photographic plate, while galaxies of low surface brightness (LSB) look small because one cannot distinguish them from the noise due to the non zero sky brightness. This effect limits strongly the ability to recognize galaxies of different surface brightness (Disney & Phillips, 1987). Surveys of LSB galaxies (Binggeli et al, 1990; Eder et al., 1989; Thuan, 1987) have shown that dwarf and LSB galaxies fall into the structures delineated by the luminous ones and that there are not evidences that these galaxies fill voids. However this effect can be relevant in the determination of number of galaxies for unit magnitude (number-count relation); in fact McGaugh (1994) stressed that, due to this selection bias, one underestimates the number of LSB in local surveys (see *section 4.1.4.*) while it does not happen in deeper samples.

Different evolution effects could be treated as an intrinsic physical selection effects. In this case we must add supplement terms in the theoretical predictions to take into account evolution (see e.g., Yoshii & Takahara, 1988; Guideroni & Rocca-Volmerange, 1990; Yoshii, 1993). The relativistic beaming effect is another example of intrinsic physical selection effect. It must be taken into account for active galaxies which contain plasma jets with relativistic bulk motion. The effect has a strong influence on measured fluxes, angular sizes and number counting of objects (see e.g. Padovani & Urry, 1992; Baryshev & Teerikorpi, 1995). The gravitational lensing is an intervening physical selection effect which easily shifts observable relations for fluxes and angular sizes. The detailed description of the gravitational lensing and its modern applications may be found in the review articles of Blanford & Narayan (1992), Refsdal & Surdej (1994), and in the monograph of Schneider et al.(1992) (see *section 4.1.5.3.*).

These examples demonstrate that the check of cosmological theories with observations, and the rejection of some of them, is not an easy task. It needs great caution, because one must take into account all the essential selection effects. The evolution, for example, has been sometimes invoked to explain the difference between theoretical predictions and observational data (e.g. in the case of number counts), but there is not a self consistent model of evolution that takes into account different experimental facts (Yoshii, 1993). Moreover, if one defines evolution as the difference between observable and theoretical quantities, one must be sure that the theoretical model has been widely verified. That is why different cosmological alternatives should be closely studied to derive more definite predictions.

### 3 THEORETICAL PREDICTIONS OF COSMOLOGICAL MODELS

The development of cosmology is basically determined by developments of the observational tests of world models. Unfortunately, in spite of a great growth of observational information, there is no agreement between the different cosmological tests and one has to add to the models new free parameters. Furthermore, the last years have generated more critical attitudes towards the standard cosmological model. From many recent redshift surveys (*section 3.4*) it follows that the length  $\lambda_0$ , above which the distribution should be smooth and essentially structureless, is not identified and that it is much greater than the previously considered size of about  $10Mpc$ .

In particular the most important assumption that has to be reconsidered is the homogeneity of matter distribution at least up to a scale 10 times larger than the previously believed scale. In many cases this situation gives rise to huge problems for the standard cosmology.

An important problem of the Standard Model is the perfect linearity of the redshift-distance relation, that is, in the traditional interpretation, a consequence of homogeneity, deeply inside the inhomogeneity cell, i.e. for distances smaller than  $\lambda_0$  (Baryshev, 1994a). The fact that there is no generally accepted clear observational determination of the Universe geometry within the framework of the standard Big Bang theory, while there are tremendous collections of observational data, suggests that we have to consider more carefully alternative cosmological theories. We shall consider in the following possible cosmological theories and their predictions for observations.

### 3.1 Paradoxes of Newtonian cosmology and origin of modern cosmological ideas

The Newtonian cosmology deals with the Euclidean space and the Newtonian theory of gravitation (for historical references see the excellent book by North, 1965). At the end of the last century, three paradoxes of the Newtonian cosmology had been formulated: Neumann-Seeliger gravitational paradox, Cheseaux-Olbers photometric paradox and Boltzmann thermodynamic paradox.

The *gravitational paradox* is the following: as the volume of a matter distribution of finite density tends to infinity, the Newtonian gravitational potential at any point has no definite value and the gravitational force also becomes indefinite. To put it in another way, the solution of the Poisson equation inside the homogenous ball is:

$$\Phi(r) = -2\pi G\rho_0(R^2 - \frac{r^2}{3}) \quad (3.1)$$

where  $\rho_0$  is the mass density,  $R$  is the radius of the ball,  $r$  is the radial distance from the center of the ball. Hence the gravitational potential in the center of the ball is:

$$\Phi(0) = -2\pi G\rho_0 R_{R \rightarrow \infty}^2 \rightarrow -\infty \quad (3.2)$$

i.e. it has an infinite value for infinite radius of the ball. The gradient of the gravitational potential at the surface of the ball is also infinite, i.e. it leads to an infinite gravity force. In terms of modern field theory, it means infinite energy density of the gravitational field, because the latter is  $(d\Phi/dr)^2/8\pi G$ .

The *background radiation paradox* is the following: if an uniform distribution of stars covers the whole sky up to some finite radius  $r$ , the night sky should be as bright as the mean star surface. If  $n_0$  is the mean number density of stars in space, then the sky fraction  $f(r)$  (dilution factor) covered by stars is:

$$f(r) = \frac{1}{4\pi} \int_0^r \frac{A}{r^2} n_0 4\pi r^2 dr \quad (3.3)$$

where  $A = \pi R_*^2$  is the surface of the star cross section and  $\Omega_* = A/r^2$  is the solid angle of the star at distance  $r$ . Therefore the sky will be completely covered ( $f(r) = 1$ ) if the stars are distributed up to the radius:

$$r_{ph} = \frac{1}{An_0} \quad (3.4)$$

The total energy density of the background radiation observed from the Earth is:

$$\rho_{BR}(r) = \frac{1}{c} \int_0^r \frac{L_*}{4\pi r^2} n_0 d\vec{r} = \frac{L_* n_0 r}{c} \quad (3.5)$$

where  $L_*$  is the typical luminosity of a star. For  $r = r_{ph}$  (Eq.3.4)  $\rho_{BR}(r)$  equals the value at the surface of a star, and from Eq.3.5:

$$\rho_{BR}(r_{ph}) = \frac{L_*}{cA} \quad (3.6)$$

The *thermodynamic paradox* is the thermal death of the Boltzmanian Universe (elastic interacting molecules), i.e. the existence of an asymptotic homogeneous state with small thermodynamics fluctuations. On the contrary we observe a highly inhomogeneous Universe with evolution of rather complex systems.

These paradoxes have been of great importance in cosmological researches because they stimulated very active search for self consistent models of the Universe (for historical background see North, 1965; and for a modern point of view see Harrison, 1981). In order to resolve a paradox, one has to take into account the initial postulates of the model used. For example for the gravitational paradox resolution one may consider some modification of the Newtonian theory of gravitation. The first solution of the gravitational paradox was found by Neumann and Seeliger (see North, 1965). They proposed that gravity falls off at large distances faster than the inverse-square law. For the gravitational potential they took the expression of the usual Newtonian form multiplied by an additional factor  $e^{-\alpha r}$ , where  $\alpha$  is sufficiently small to be consistent with the usual Newtonian theory for small distances. This makes possible an infinite quasi-Newtonian Universe without gravitational paradox. In fact, the same idea was used by Einstein (1917), who introduced the  $\Lambda$ -term in General Relativity. Instead of the Poisson equation he suggested:

$$\Delta\phi - \Lambda\phi = 4\pi G\rho_0 \quad (3.7)$$

where  $\lambda$  is the universal constant. The solution of Eq.3.7 is:

$$\phi = -\frac{4\pi G}{\Lambda}\rho_0 = const \quad (3.8)$$

It corresponds to an infinite distribution of homogeneous matter which is in equilibrium with internal forces. Local inhomogeneities in matter distribution will add a local potential  $\phi$  which will be like the Newtonian one for sufficiently large  $4\pi G\rho_0$  relative to  $\Lambda\phi$ . In *section 3.4.7.* it will be shown that this seemingly "ad hoc" hypothesis has a very clear foundation within the framework of the tensor field gravitation theory.

The second idea which has been recently used in cosmological papers again, is the hierarchical distribution of matter (known now as fractal). It was shown by Fournier D'Albe (1907) and Charlier (1908, 1922) that the Newtonian Universe may be build up without gravitational and photometric paradoxes if the matter is distributed according to the unlimited clustering hierarchy. In the model the typical values of the mass within the distance  $r$  of hierarchy scale as:

$$M(r) \approx r^1 \quad (3.9)$$

The size and the mass of the Universe are arbitrarily large, but the mean density  $M(r)/r^3 \approx r^{-2}$  converges to zero and Olbers's paradox is avoided. The virial velocity in a cluster of size  $r$  is  $v^2 \approx M/r = const$ , independent of  $r$ . The modern development of this idea will be discussed in *section 3.4* and the modern observational data which confirm the fractal structure of matter distribution at least up to  $100Mpc$  will be considered in *section 3.4.3., 3.4.4., 4.2.3.*

The third basic idea in modern cosmology was the expanding Universe model. The idea was a consequence of the geometrical revolution in gravitation theory, which had been made by Einstein's General Relativity (Einstein, 1916). Non static solutions of Einstein's equations (Friedmann, 1922) had shown that the Universe can expand or contract in the sense of continuous space creation

or annihilation. This idea has been developed by many authors and it is one of the main initial hypotheses in the most modern cosmological models.

The next important idea was Milne's discovery of a purely Newtonian derivation of the Friedmann equations (Milne, 1934; McCrea & Milne, 1934). Milne and Mc Crea showed that the expanding Universe equations, which had been previously derived from General Relativity, can be obtained directly from simple Newtonian theory (using flat space, static Euclidean space, Newtonian time and Newtonian dynamics). All one has to do is to consider a ball of matter with finite radius  $R$  ("cosmic ball") and forget about the matter outside the ball. Then the radius  $R$  of the expanding spherical symmetric cosmic ball will be governed by the equation:

$$\frac{d^2 R}{dt^2} = -\frac{4\pi G}{3} R \rho \quad (3.10)$$

or after integrating

$$\left(\frac{dR}{dt}\right)^2 = -\frac{8\pi G}{3} R^2 \rho - k v_0^2 \quad (3.11)$$

where constant  $k$  has the value 0, 1 or  $-1$ , which corresponds to parabolic, elliptic or hyperbolic motion of the ball's particles, and  $v_0^2$  is the module of the integration constant. This discovery has created a problem in cosmology: How can it be that the non-relativistic Newtonian theory yields exact general relativistic results? Where are such restrictions as the limit on the velocity of light and general retarded response effects?

A very important idea is the suggestion made by Gamow, Ivakenko and Landau (1928) and by Bronstein (1934) about the relativistic-quantum-gravity, ( $Ghc$ ), character of the future cosmological theory. In this approach, gravitation theory must be a quantum relativistic field theory, because it is based on the existence of the quanta of the field, i.e. gravitons, which carry energy-momentum in space and may transform into other elementary particles (Ivanenko & Sokolov, 1947). Modern attempts to join quantum field particle physics and General Relativity (hereafter GR) (such as inflation and quantum cosmology) continue these ideas.

Another cosmological idea that needs to be mentioned is the absorber theory of Wheeler and Feynman (1949). In this theory local and global Universe could be described as a single system. The theory is free from an action of an elementary charge upon itself and provides an experimentally satisfactory account of the behaviour of a system of point charges in electromagnetic interaction with one another. Cosmological implications of the theory have been considered by Hoyle & Narlikar (1974).

Now we shall consider how the ideas mentioned above work in different cosmological models.

## 3.2 Big Bang models

The Hot Big Bang (HBB) scenario is the currently most accepted model so that one refers to it as the "Standard Model". A complete introduction to this model can be found in the books of Weinberg (1972) and of Peebles (1980; 1993), while discussions on its observational tests and its state of art are found in the reviews of Sandage (1988), Peebles et al. (1991; 1994), Coles & Ellis (1994).

### 3.2.1 Initial Hypotheses of the standard Friedmann model

The *first hypothesis* of the Standard Friedmann model (hereafter SM) is that GR is a correct relativistic gravitation theory, and that it can be applied to the Universe as a whole. According



to this hypothesis gravity is described by a metric tensor  $g^{ik}$  of a Riemannian space. The "field" equations of GR (Einstein-Hilbert equations) have the form:

$$R^{ik} - \frac{1}{2}Rg^{ik} = \frac{8\pi G}{c^4}T_{(m)}^{ik} \quad (3.12)$$

where  $R^{ik}$  is the Ricci tensor,  $T_{(m)}^{ik}$  is the energy-momentum tensor (hereafter EMT) for matter only. Solutions of the Eq.3.12 for unbounded matter distribution are cosmological ones and are the basis of all cosmological interpretations.

The *second hypothesis* is the homogeneity of the matter distribution in space, i.e.:

$$\rho(\vec{r}, t) = \rho(t) \quad (3.13)$$

$$p(\vec{r}, t) = p(t) \quad (3.14)$$

Peebles (1980) discussed in detail if the homogeneity of the Universe has to be expected from general physical arguments. He emphasized that within the SM we cannot account for the homogeneity and we must accept it as an assumption (see also Harrison, 1981). This means that homogeneity must be accepted as a phenomenon to be explained by some future deeper theory. On the other hand, modern observations of the large-scale structure of the Universe point to the existence of very large fractal like inhomogeneities. Hence the homogeneity assumption can be tested experimentally and it can be postulated only for sufficiently large scales (see *section 4.2.3.* and *section 3.4*).

From the assumption of homogeneity in Friedmann cosmology it follows also the isotropy of the matter distribution. The requirement for Isotropy and Homogeneity is implemented by the Cosmological Principle in order to avoid a privileged observer. We show in *section 3.4.1.* that this last condition is ensured only by local isotropy and the requirement of homogeneity is a very strong one and it is not necessary in order to avoid any privileged observer in the Universe.

As a consequence of homogeneity and isotropy one can get the Robertson-Walker line element in the form:

$$ds^2 = c^2 dt^2 - R(t)^2 [dr^2 + I(r)^2 (d\theta^2 + \sin^2 \theta d\phi^2)] \quad (3.15)$$

where  $r, \theta, \phi$  are the comoving space coordinates,  $I(r) = \sin(r), r, \sinh(r)$  corresponding to curvature constant values  $k = +1, 0, -1$  respectively and  $R(t)$  is the scale factor. The *expanding space paradigm* is that the *metric* (proper) *distance* of a comoving body of fixed coordinate distance  $r$  from a comoving observer is:

$$l = R(t) \cdot r \quad (3.16)$$

and increases with time  $t$  as the scale factor  $R(t)$ . Note that physical dimension of proper distance  $[l] = cm$ , hence if  $[R] = cm$  then  $r$  is a dimensionless coordinate distance.

The recession "velocity", or the space expansion velocity, of a comoving (i.e., with  $r$  constant) body is:

$$V = \frac{dl}{dt} = \frac{dR}{dt} r = \frac{dR}{dt} \frac{l}{R} = H(t)l = c \frac{l}{R_H} \quad (3.17)$$

where  $H(t) = (1/R)dR/dt$  is the rate of the expansion (Hubble constant) and  $R_H = c/H(t)$  is the Hubble distance at the time  $t$ . The "velocity"  $V$  - distance  $l$  relation (Eq.3.17) is linear for all distances  $l$ : therefore for  $l > R_H$  we get  $v > c$  (Harrison, 1993). In connection with this the quantity  $V$  would be better called the "*space expansion velocity*", because it is not the usual physical velocity of a body in space. The expansion of space induces the wave stretching of the travelling photons via the Lemaitre's equation (Eq.2.5), i.e.:

$$(1 + z) = \frac{\lambda_0}{\lambda_1} = \frac{R_0}{R_1} \quad (3.18)$$

where  $\lambda_1$  and  $\lambda_0$  are the wavelengths at the emission and reception respectively, and  $R_1$  and  $R_0$  the corresponding values of the scale factor. Equation 3.18 may be obtained from the radial null geodesic ( $ds = 0$ ,  $d\theta = 0$ ,  $d\phi = 0$ ) of the Robertson-Walker line element Eq.3.15.

The behavior of the scale factor with time  $R(t)$  is governed by the Einstein-Hilbert equations (Eq.3.12) which, in the case of homogeneity, gives the Friedmann equation:

$$\frac{d^2 R}{dt^2} = -\frac{4\pi G}{3} R \left( \rho + \frac{3p}{c^2} \right) \quad (3.19)$$

or after integrating

$$\left( \frac{dR}{dt} \right)^2 = -\frac{8\pi G}{3} R^2 \rho - kc^2 \quad (3.20)$$

where  $k = 0, +1, -1$  for flat, closed and open space geometry. Solving this equation we find the dependence of the scale factor from time, i.e.  $R(t)$ . The model has two parameters that are the Hubble parameter:

$$H = \frac{dR}{dt} \frac{1}{R} \quad (3.21)$$

and the deceleration parameter:

$$q = -R \frac{d^2 R}{dt^2} \left( \frac{dR}{dt} \right)^{-2} \quad (3.22)$$

which, for the present time  $t_0$ , are  $H(t_0) = H_0$  and  $q(t_0) = q_0$  respectively. In this theory we have also the density parameter:

$$\Omega = \frac{\rho}{\rho_c} \quad (3.23)$$

$$\rho_c = \frac{3H^2}{8\pi G} \quad (3.24)$$

and the space curvature parameter:

$$K = \frac{kc^2}{(HR)^2} \quad (3.25)$$

These parameters satisfy the equations:

$$\Omega = K + 1 \quad (3.26)$$

and

$$q = \frac{1}{2} \Omega \left( 1 + \frac{3p}{\rho c^2} \right) \quad (3.27)$$

Thus for  $p \ll \rho c^2$  the SM is fixed by two parameters  $\Omega$  and  $H$  or  $q$  and  $H$ . Eq.3.27 means that dynamics and geometry of the model are uniquely defined. From Eq.3.26 it follows that

$$H(z) = H_0(1+z)(1+z\Omega_0)^{1/2} \quad (3.28)$$

and it may be shown that Eq.3.17 can be written in the form (Harrison, 1993):

$$V = c(\Omega_0 - 1)^{-1/2} \sin^{-1} \left( \frac{2(\Omega_0 - 1)^{1/2}}{\Omega_0^2(1+z)} \left( z\Omega_0 + (\Omega_0 - 2)(1+z\Omega_0)^{1/2} - 1 \right) \right) \quad (3.29)$$

In the limit of  $z \ll 1$  this yields:

$$V \approx cz \quad (3.30)$$

The *third hypothesis* of the SM is that the observed redshift in Eq.3.30 is due to the space expansion, i.e. given by Eq.3.18, and that the real distance equals the proper distance in Eq.3.16. Hence for  $z \ll 1$  and  $l \ll R_H$  from Eq.3.30 and Eq.3.17 we get the relation:

$$z \approx \frac{V}{c} = \frac{H_0}{c} l = \frac{l}{R_H} \quad (3.31)$$

which may be interpreted as the observable Hubble relation. We stress again that the quantity  $V$  is not the velocity of a receding galaxy in the usual sense, but  $V$  is the velocity of the space expansion ("creation of space") and such a phenomenon has never been tested in laboratory physics.

In the SM it is suggested (*the fourth hypothesis*) that the Universe has expanded from a very hot state, and due to the creation of space the matter cools as in usual thermodynamics. Note that as in the case of the expansion velocity, in this case we have no "usual" thermodynamics, because the equation  $dU = -pdV$  is laboratory tested only for changes of the gas volume  $dV$  in the static Euclidean space without space creation.

### 3.2.2 Successes of the standard model

There are several great successes of the application of the SM to the real observed Universe. Review of the evidences in favor of the relativistic HBB models can be found, for example in Weinberg, 1972; Sandage, 1987, 1988; Peebles, 1980, 1993; Peebles et al. 1991, 1994. There are no gravitational, photometric and thermodynamic paradoxes in the SM, because the age of the Universe is finite and rather small in comparison with any reasonable time scale. In the SM, space has been filled with blackbody radiation, the cosmic microwave background radiation (CMBR). As the Universe expands, the number of CMBR photons per unit volume drops as

$$n_0 = \frac{n(z)}{(1+z)^3} \quad (3.32)$$

when  $n_0$  is the present value of the number density. The photon wavelength is stretched by the expansion as in Eq.3.18. Thus the CMBR preserves a black body spectrum with a temperature that decreases as the Universe expands

$$T_0 = \frac{T(z)}{(1+z)} \quad (3.33)$$

where  $T_0$  is the present temperature of the CMBR and its observed value is about  $3K$ . Eq.3.32 and Eq.3.33 can be used back to  $z \sim 10^{10}$ . The observed thermal spectrum of the CMBR is the greatest success of the SM predictions, even if its fundamental quantity,  $T_0$ , cannot be deduced from any calculations of the early Universe. We stress that the SM determines only one of the three characteristics of the CMBR: the blackbody spectrum. In fact, as we shall see in *section.3.2.5*, the extreme isotropy of CMBR is a paradox for the SM.

In the SM, the Universe was hot and dense enough to drive thermonuclear reactions that changed the chemical composition of the matter. The values of the abundances left over from this hot epoch depend on the cosmological parameters. Knowing the present temperature and assuming a value for the present matter density, we have fixed the thermal history of the Universe. If the matter is uniformly distributed and lepton members are comparable to the baryon number, this is sufficient to fix the final abundances of the light elements. The observed light element abundances of  ${}^4He$ ,  ${}^2H$ ,  ${}^3He$  and  ${}^7Li$  are in good agreement with SM predictions, provided that the baryonic density is in a defined and narrow range. In particular the abundance of  $He$  and  $D$  are explained with high accuracy, while there are more uncertainties in the abundances of  $Li$ ,  $Be$  and  $B$  (Boesgaard & Steigman, 1985).

### 3.2.3 Crucial Tests

According to our definition (see *section 2.3*), crucial tests concern the validity of the different cosmological theories. Therefore crucial tests for SM will be the following:

- experimental testing of General Relativity
- determination of the matter distribution in space
- testing of the reality of space expansion
- measuring the temperature of the CMBR at high redshift
- determination of the ages of the oldest objects
- measuring the evolution of the chemical composition of matter at high redshift

We shall consider the tests in *section 4.2*.

### 3.2.4 Parametric tests

The most reliable observational data have been collected for classical cosmological tests. Here we discuss the predicted relations among observable quantities in the framework of the standard Friedmann model (for more details see e.g. Weinberg, 1972; Sandage, 1987, 1988; Yoshii & Takahara, 1988). In *section 4.1*, we consider the comparison of these predictions with observational data.

#### 3.2.4.1 The proper distance-redshift relation

The basic relation for the calculation of different observable quantities is the connection between metric (proper)  $l$ , angular  $l_a$  and bolometric  $l_{bol}$  distances in the expanding Universe:

$$l = l_a(1 + z) = \frac{l_{bol}}{(1 + z)} \quad (3.34)$$

For metric distance (Eq.3.16) we have

$$\mathcal{L}(z, q_0) \equiv \frac{l}{R_{H_0}} = \frac{zq_0 + (q_0 - 1)((2q_0z + 1)^{1/2} - 1)}{q_0^2(1 + z)} \quad (3.35)$$

where  $R_{H_0} = c/H_0$ . Note that for the case  $q_0 = 1/2$  and  $z \ll 1$  from Eq.3.17 we get:

$$z_{cos} = \frac{v}{c} + \frac{1}{2} \left( \frac{v}{c} \right)^2 + \frac{1}{2} \frac{GM(l)}{c^2 l} = z_D + z_{grav} \quad (3.36)$$

where we used the relation  $x \sim z \sim v/c$  and the energy equation  $v^2 = 2GM(l)/l$ . So the cosmological redshift can be considered as the sum of the Doppler and the gravitational parts, at least, in the small  $z$  approximation.

#### 3.2.4.2 The angular size-redshift relation

From Eq.3.34 and Eq.3.35 we get theoretical angular size-redshift relation in the form:

$$\Theta(z, q_0) = \left( \frac{d}{R_{H_0}} \right) \left( \frac{\mathcal{L}(z, q_0)}{(1 + z)} \right)^{-1} \quad (3.37)$$

where  $\Theta$  is the angular size of the body with linear size  $d$ , and the  $\mathcal{L}$  function is given by Eq.3.35.

### 3.2.4.3. The magnitude-redshift relation

Let  $L(\nu_{emit})d\nu_{emit}$  be the spectral luminosity between  $\nu_{emit}$  and  $\nu_{emit} + d\nu_{emit}$  emitted from a source at redshift  $z$ . Then the observed luminosity will be:

$$L(\nu_{obs})d\nu_{obs} = \frac{L(\nu_{emit})d\nu_{emit}}{(1+z)^4} = \frac{L(\nu_{obs}(1+z))}{(1+z)^3}d\nu_{obs} \quad (3.38)$$

The apparent bolometric flux received at Earth from a source with redshift  $z$ , with its absolute bolometric luminosity

$$L_{bol} = \int L(\nu)d\nu \quad (3.39)$$

is given by

$$S_{bol} = \frac{L_{bol}}{4\pi l_{bol}^2} = \frac{L_{bol}}{4\pi R_{H_0}^2 \mathcal{L}^2(z, q_0)(1+z)^2} \quad (3.40)$$

Converting Eq.3.40 into magnitude one obtains the theoretical magnitude-redshift  $m(z)$  relation

$$m_{bol} = 5 \log(R_{H_0} \mathcal{L}(z, q_0)(1+z)) + 25 + M_{bol} \quad (3.41)$$

where  $\mathcal{L}$  is given by Eq.3.35,  $R_{H_0}$  is measured in Mpc,  $M_{bol}$  is the bolometric absolute magnitude of the source. For  $z \ll 1$  we get

$$m_{bol} \approx 5 \log z + 1.086(1 - q_0)z + const. \quad (3.42)$$

The first term in the right side of Eq.3.42 is the usual Hubble  $m(z)$ -diagram and the second is the cosmological correction. In the experimental astrophysics, a fixed detector effective wavelength " $i$ " and a finite bandwidth  $\zeta_i$  are used. Then the flux in the filter  $i$  is given by

$$S_i = \int S_\nu d\nu = \frac{L_i}{4\pi l_{bol}^2} \left( \frac{\tilde{L}_i}{L_i} \right) \quad (3.43)$$

where

$$L_i = \int L(\nu)\zeta_i(\nu)d\nu \quad (3.44)$$

$$\tilde{L}_i = (1+z) \int L(\nu(1+z))\zeta_i(\nu)d\nu \quad (3.45)$$

or in magnitudes

$$m_i = 5 \log l_{bol} + 25 + M_i + K_i(z) \quad (3.46)$$

with

$$K_i(z) = -2.5 \log \left( \frac{\tilde{L}_i}{L_i} \right) \quad (3.47)$$

$K_i(z)$  is the  $K$ -correction. In evaluating the correction at fairly large redshifts the spectral energy distribution  $L(\nu)$  and the sensitivity function  $\zeta_i$  are necessary. In the case of spectral observations the observed flux density of an object with spectral luminosity  $L(\nu)$  is given by:

$$S_{\nu_{obs}} = \frac{L(\nu_{obs}(1+z))}{(1+z)^3 4\pi l_a^2} \quad (3.48)$$

For power law luminosity  $L_{\nu_{emit}} \sim \nu_{emit}^{-\alpha}$  we have

$$S_{\nu_{obs}} = \frac{L(\nu_{obs})}{(1+z)^{1+\alpha} 4\pi l_{sp}^2} = \frac{L(\nu_{obs})}{4\pi l_{sp}^2} \quad (3.49)$$

Hence in that case the spectral distance  $l_{sp}$  is

$$l_{sp} = R_{H_0} \mathcal{L}(z, q_0) (1+z)^{\frac{1+\alpha}{2}} \quad (3.50)$$

The measured surface brightness of the source is the received flux divided by the apparent area obtained by combining Eq.3.37 and Eq.3.40, which give the bolometric one:

$$J_{bol}(z) = \frac{S_{bol}}{\Theta^2} = \frac{const}{(1+z)^4} \quad (3.51)$$

From Eq.3.40 and Eq.3.43 the filtered surface brightness is

$$J_i(z) = \frac{S_i}{\Theta^2} = \frac{const}{(1+z)^4} \cdot 10^{-0.4K_i(z)} \quad (3.52)$$

where  $K_i(z)$  is given by Eq.3.47.

#### 3.2.4.4. Time-redshift relation

In the framework of the SM the age of the Universe at redshift  $z$  is given by

$$t(z) = \frac{1}{H_0} \int_0^{\frac{1}{1+z}} \frac{dy}{(1+2q_0+2q_0/y)^{1/2}} \quad (3.53)$$

or in differential form

$$dt = -\frac{dz}{H_0(1+z)^2(1+2q_0z)^{1/2}} \quad (3.54)$$

The present age of the Universe  $t_0$  for  $q_0 = 0.5$  is  $t_0 = \frac{2}{3}H_0^{-1}$ , and for  $q_0 = 0$   $t_0 = H_0^{-1}$ . The age of an object of redshift  $z$  which was formed at  $z_f$  is given by

$$t_{obj}(z) = t(z) - t(z_f) \quad (3.55)$$

#### 3.2.4.5. Count-redshift and count-magnitude relations

Let  $n_0$  be the number density of objects in space, then the differential number counts of the objects with the redshift between  $z$  and  $z+dz$  will be

$$dN(z, q_0) = 4\pi n_0 \frac{dV}{dz} dz \quad (3.56)$$

where  $dV/dz$  is the differential comoving volume element of the standard model, which is given by:

$$\frac{dV(z, q_0)}{dz} = \frac{4\pi c^3 \mathcal{L}^2(z, q_0)}{H_0^3(1+z)(1+2q_0z)^{1/2}} \quad (3.57)$$

Eq.3.57 takes into account expansion and non- Euclidean geometry of the space. For example, for the case of  $q_0 = 0.5$  we can get integral count-redshift relation  $N(z, q_0)$  integrating Eq.3.56 over  $z \in [0, z]$  :

$$N(z, 0.5) = \frac{32\pi c^3}{3H_0^3} n_0 \left( \frac{(1+z)^{1/2} - 1}{(1+z)^{1/2}} \right)^3 \quad (3.58)$$

The  $N(z, q_0)$  relation can be transformed to  $N(m, q_0)$  using Eq.3.46. Let  $d^2A(m, z, q_0)$  be the number of objects with redshift  $[z, z+dz]$  contributing to the counts per steradians and magnitude bin around apparent magnitude  $m_i$ , i.e.:

$$d^2A(m_i, z, q_0) = \Phi(M_i) \frac{dV}{dz} dm_i dz \quad (3.59)$$

where  $\Phi(M_i)$  is the luminosity function per volume unit,  $M_i$  is the absolute magnitude through the "i" filter and is given by Eq.3.46:

$$M_i = m_i - 5 \log (\mathcal{L}(z, q_0)(1+z)R_{H_0}(Mpc)) - 25 - K_i(z) \quad (3.60)$$

The differential number count per steradian of objects in the "i" band and magnitude bin  $dm_i$  is obtained by integrating Eq.3.59 on  $z$ :

$$N_d(m_i, q_0) = \int_0^{z_{max}} d^2A(m_i, z, q_0) \quad (3.61)$$

where  $z_{max}$  is the maximum redshift for the objects. The number of points per steradian and redshift bin  $dz$  can be computed by integrating Eq.3.59 on  $m_i \in [m_1, m_2]$ :

$$N_d(z, q_0) = \int_{m_1}^{m_2} d^2A(m_i, z, q_0) \quad (3.62)$$

#### 3.2.4.6. Cosmic Background Radiation

The effective brightness of the diffuse background radiation expected from a population of sources in a volume extending from  $z = 0$  to  $z = z_{max}$  depends on the luminosity function  $\Phi(L_\nu)$  and the spectral energy distribution of the source  $L_\nu$ :

$$J_\nu = c^3 H_0^{-3} \int_0^\infty dL_\nu \int_0^{z_{max}} S_\nu(z') L^2(z', q_0) (1+z')^{-4} (1+2q_0 z')^{-1/2} \Phi(z', L_\nu) dz' \quad (3.63)$$

where  $S_\nu(z)$  is the flux density of an object at frequency  $\nu$  and is given by Eq.3.48 and

$$\Phi(z, L_\nu) = \Phi(0, L_\nu) (1+z)^3 \quad (3.64)$$

is the volume density of objects. The dimension of  $I_\nu$  is  $Jy \cdot sr^{-1} Hz^{-1}$ . The energy density of the background radiation can be expressed in the form:

$$\rho_\nu = \frac{4\pi}{c} I_\nu = \frac{L_{0\nu} n_0}{H_0} b(z_{max}, q_0) \quad (3.65)$$

where  $L_{0\nu} n_0 / H_0 = \rho_\nu$  is the characteristic radiation energy density of the fixed type of objects with mean number density  $n_0$  and luminosity density  $L_{0\nu}$  and  $b(z, q_0)$  is the model dependent factor.

### 3.2.5 Paradoxes of the standard model

The Standard Hot Big Bang model has been accepted by most physicists. We shall confront the SM with modern observational data in *section 4*. Here we discuss several paradoxes of the SM. For a more detailed description of these paradoxes see e.g. von Horner, 1974; Guth, 1992; Linde et al., 1994; Blau & Guth, 1987; Harrison, 1993; 1995; Baryshev, 1994a).

i.) The first of these paradoxes is called the *flatness paradox*. The behavior of  $(\Omega - 1)/\Omega$ , with time, can be written as

$$\frac{\Omega - 1}{\Omega} = \frac{3k}{8\pi G\rho R^2} \quad (3.66)$$

Then one can calculate the allowed range of  $\Omega$  at the Planck time  $t_{pl} = (Gh/c)^{1/2} = 5.4 \cdot 10^{-44} \text{ sec}$

$$|\Omega - 1| < 10^{-59} \quad (3.67)$$

The flatness problem is then the difficulty in understanding why  $\Omega$  was so close to one, while  $\Omega = 1$  is a set of measure zero on the real line of all possible values of  $\Omega$ . The question is: why is the Universe close to flat, i.e. why is the geometry of the real space nearly Euclidean ( $k = 0$ ) ?

ii.) The second paradox is the *isotropy paradox*. The problem is related to the large-scale isotropy of the observable Universe, seen most strikingly in the isotropy of the CMBR. In the context of the standard model the horizon distance, i.e. the distance that a light pulse could have traveled since the singularity at the time  $t = 0$ , is  $r = ct$ , while the scale factor is  $R(t) \sim t^{1/2}$  (for early stages). Hence the ratio  $R/r \sim t^{-1/2} \rightarrow \infty$  for  $t \rightarrow 0$ . At the time of recombination  $t_r$  two emitters  $A$  and  $B$  of CMBR photons, arriving at the earth today from two opposite directions in the sky, are separated from each other by more than 70 times the distance that light could have traveled up until that time. Thus, there is no way that a point  $A$  and  $B$  could have communicated with each other, and no physical process that would bring them to the same temperature. The paradox is that the large-scale isotropy of the Universe, proved by the isotropy of the CMBR, cannot be explained by the SM and it must be assumed as an initial condition.

iii.) The third paradox is the *superluminal velocity paradox*. The velocity of space expansion  $V \sim dR(t)/dt \sim t^{-1/2}$  and  $V \rightarrow \infty$  for  $t \rightarrow 0$ . Hence as noted by Von Horner (1974) and Murdoch (1977), the SM permits superluminal motions of any distinct point at early epoch. More recently Harrison (1993) has analyzed the redshift-distance and the velocity-distance relations and concluded that the linear  $V(t) = c \cdot l/R_H$  law applies quite generally in expanding homogeneous and isotropic cosmological models, and the recession velocity  $V$  can exceed the velocity of light if  $l > R_H$  (see Eq.3.17). This violates the premises of Special Relativity, but it is permitted by GR. In GR space is not rigid and can bend, twist and stretch; there is nothing in General Relativity that places any limit on the speed with which such stretching can take place (Guth, 1992). However the paradox is that the permanent space creation demands the violation of the maximum motion velocity principle, because in this case we get information about increasing volume of space via local physical parameters, such as the matter density.

iv.) The fourth paradox is the *inhomogeneity paradox*. In the framework of the SM the observed linear Hubble law ( $z \sim l^1$ ) is a consequence of the homogeneity of matter distribution. It is not clear from observation at which scale the cut-off towards homogeneity is reached (see *section 4.2.3.*, *3.4.3.*), but it is evident that at small scales, i.e. for distances lower than  $\sim 100 \text{ Mpc}$ , the distribution of matter is fractal. It was shown by Haggerty & Wertz (1972), Fang et al. (1991) and Ribeiro (1992a, 1992b, 1993) that density fluctuations inside the fractal inhomogeneity cell will lead to strong disturbance of pure Friedmann behavior. However, observations suggest the opposite conclusion: according to Sandage (1986; 1994) a striking linearity of the  $z - l$  relation is observed in the distance range  $(2 - 25) \text{ Mpc}$  (see *section 4.1.1.*). This is the inhomogeneity paradox: a highly inhomogeneous galaxy distribution at scales where the  $z - l$  relation is linear means that the Hubble law should not be a consequence of homogeneity (see Baryshev, 1994a).

v.) The fifth paradox is the *global energy paradox*. If one divides the Universe in "cosmic boxes" or "cells" which are representative samples of the Universe, then what is inside is always in the same state as what is outside. Hence the Universe is not like a steam engine and pressure is not the cause of expansion. The Universe has no edge and the pressure everywhere is therefore



impotent and unable to produce mechanical energy. As was emphasized by Harrison (1981): "the conservation of energy principle serves us well in all science except in cosmology" and "the total energy decreases in an expanding Universe and increases in a collapsing Universe. To the question where the energy goes in an expanding Universe and where it comes from in a collapsing Universe the answer is - nowhere, because in this case energy is not conserved." Hence due to the creation of space the total energy in the Universe is not conserved (see Harrison, 1995).

### 3.2.6 Non-standard models

There are several modifications of the SM. Among these, we can mention the cosmological constant theory, inflationary cosmology, rotation of the Universe and others. Now we consider only the first two which are the more developed models.

#### 3.2.6.1. $\Lambda$ - term

In General Relativity, the gravitational field equations may be written as :

$$R^{ik} - \frac{1}{2}g^{ik}R - g^{ik}\Lambda = \frac{8\pi G}{c^4}T_{(m)}^{ik} \quad (3.68)$$

where the  $\Lambda$  is the famous cosmological constant. During the last twenty years the  $\Lambda$ -term has appeared and disappeared repeatedly in cosmological papers. Note that Einstein called it "*the biggest blunder of my life*". Now the  $\Lambda$ -term is popular again, because it gives additional possibilities to fit observational data (see e.g. Carroll et al., 1992; Croswell, 1993). Instead of Eq.3.26 and Eq.3.27 we get (for  $p \ll \rho c^2$ ):

$$\Omega - \lambda = K + 1 \quad (3.69)$$

$$q = \frac{1}{2}\Omega - \lambda \quad (3.70)$$

where  $\lambda = \Lambda c^2 / (3H^2)$ . The bolometric distance  $l_{bol}$  is given by:

$$l_{bol} = \frac{c(1+z)}{H_0} \left[ \frac{\frac{1}{(-K_0)^{1/2}} \sinh \left( \int_{\frac{1}{1+z}}^1 \frac{(-K_0)^{1/2} dy}{y(\Omega_0/y - K_0 + \lambda_0 y^2)^{1/2}} \right)}{\int_{\frac{1}{1+z}}^1 \frac{dy}{y(\Omega_0/y + \lambda_0 y^2)^{1/2}}} + \frac{\frac{1}{(K_0)^{1/2}} \sin \left( \int_{\frac{1}{1+z}}^1 \frac{(K_0)^{1/2} dy}{y(\Omega_0/y - K_0 + \lambda_0 y^2)^{1/2}} \right)}{\int_{\frac{1}{1+z}}^1 \frac{dy}{y(\Omega_0/y + \lambda_0 y^2)^{1/2}}} \right] \quad (3.71)$$

for  $k = -1$ ,  $k = 0$ ,  $k = +1$  respectively. In terms of  $l_{bol}$ , the angular distance  $l_a$  and the comoving volume element  $dV/dz$  are expressed respectively as :

$$l_a = l_{bol} / (1+z)^2 \quad (3.72)$$

$$\frac{dV}{dz} = \frac{4\pi c l_{bol}^2}{H_0(1+z)^3(\Omega_0(1+z) - K_0 + \lambda_0/(1+z)^2)^{1/2}} \quad (3.73)$$

The age of the Universe at redshift  $z$  is given by:

$$t(z) = \int_0^{\frac{1}{1+z}} \frac{dy}{(\Omega_0/y - K_0 + \lambda_0 y^2)^{1/2}} \quad (3.74)$$

A Universe with a cosmological constant can behave in different ways than SM allows. There is a loiter phase which occurs when the cosmological constant  $\Lambda$  and the mass density  $\Omega$  nearly balance each other. The loiter phase can last so long that the Universe can be far older than the ages that comes from standard calculation based on its rate of expansion and mass density. Moreover the present age of the Universe depends upon the  $\Lambda$ -constant and can be compatible with the current estimates (*section 4.1.1.*) for an appropriate choice of  $\Lambda$ .

Theoretical expectations for the cosmological constant exceed observational limits by some 120 orders of magnitude. This is because anything that contributes to the energy density of the vacuum acts just like a cosmological constant. In fact in the vacuum the EMT can be written as:

$$T_{\mu\nu} = < \rho > g_{\mu\nu} \quad (3.75)$$

so that the effective cosmological constant can be written as (Eq.3.68)

$$\Lambda_{eff} = \Lambda + 8\pi G < \rho > \quad (3.76)$$

Then the cosmological constant contributes to the total vacuum energy with a term

$$\rho_v = < \rho > + \frac{\Lambda}{8\pi G} = \frac{\Lambda_{eff}}{8\pi G} \quad (3.77)$$

A crude experimental upper bound on  $\Lambda_{eff}$  is provided by the value of the Hubble constant, the flatness of space and the mass density of the Universe (Weinberg, 1989; Carroll et al., 1992)

$$|\rho_v| < 10^{-47} GeV^4 \quad (3.78)$$

The energy density of the empty space is the sum of the zero point energies of the normal modes of some field up to a wave number cutoff  $k_{max}$  ( $\hbar = c = 1$ ):

$$< \rho > \sim \frac{k_{max}^4}{16\pi^2} \quad (3.79)$$

If we take the Planck mass  $k_{max} \sim (8\pi G)^{-1/2}$  from Eq.3.79 it follows that  $< \rho > \sim 2 \cdot 10^{71} GeV^4$  so that there is a difference of 118 times with the observational upper limit (Eq.3.78). If we take the zero point energy of quantum chromodynamics  $< \rho > \sim 10^{-6} GeV^4$  the term  $\Lambda/(8\pi G)$  in Eq.3.77 cancels this term to about 41 decimal places ! Weinberg in his review (1989) considers various possible solutions of this problem based on different approaches: all these approaches show that the cosmological constant problem has great impact on other areas of physics or astronomy. Weinberg notes: "More discouraging than any theorem is the fact that many theorists have tried to invent adjustment mechanisms to cancel the cosmological constant, but without any success so far" .

### 3.2.6.2. Inflationary Universe

The inflationary model was proposed by Guth (1981) to avoid the flatness and the isotropy paradoxes. There are several modifications of the inflation scenario, which improve Guth's original model (see e.g. Linde, 1990; Guth, 1992). The main features of the various inflationary universe scenarios is the existence of some stage of evolution at which the universe expands exponentially, while it is in a vacuum-like state containing some classical homogenous fields but (almost) no particles. After the inflation the vacuum-like state decays into particles and heats up the Universe; from then the evolution can be described by the standard Hot-Big-Bang theory. The homogenous classical field responsible for inflation is present in all the Grand Unified Theories (GUT) of elementary particles, and can play the role of an unstable vacuum state.

If the Universe has ever been dominated by a false (unstable) vacuum characterized by a pressure  $p = -\rho \cdot c^2$ , then the pressure term in the Friedmann Eq.3.19 has negative sign, and overcomes the gravitational attraction caused by the usual energy density term. The force of gravity actually becomes repulsive and the scale factor has an exponential behavior:

$$R(t) = R_0 e^{(\chi t)} \quad (3.80)$$

where:

$$\chi = \left( \frac{8\pi G \rho_{vac}}{3} \right)^{1/2} \quad (3.81)$$

and  $\rho_{vac}$  is the mass density of the vacuum at time  $t$ . The precise value of the  $\rho_{vac}$  is not well constrained, and for a typical GUT  $\rho_{vac} \sim (10^{14} GeV)^4 / (\hbar c)^3 \approx 10^{74} gr \cdot cm^{-3}$  (Blau & Guth, 1987), while it also has been used the value of the Plank density  $\rho_{vac} \sim (10^{19} GeV)^4 / (\hbar c)^3 \approx 10^{94} gr \cdot cm^{-3}$  (Novikov, 1988).

Thus the Universe expands exponentially due to the gravitational repulsion of this false vacuum. During the inflationary period, the density of any particles that may have been present before inflation, is diluted so much that it becomes completely negligible. The isotropy paradox is solved by an enormous expansion: the process of inflation magnified very small regions to become large enough to encompass the entire observed Universe. Thus, the emitters of the background radiation arriving today from opposite directions in the sky, had time to reach a common temperature during the inflationary era. The flatness paradox is solved because the inflationary process determines a density  $\rho = \rho_{crit}$ , so that after inflation and vacuum decay one gets  $\Omega = 1$ , or in the case of non-zero  $\Lambda$ -term  $\Omega + \Lambda = 1$ . Thus the inflation model predicts large-scale isotropy, homogeneity and flatness of the Universe. Moreover during inflation, from the quantum fluctuations of the scalar fields, could be generated the adiabatic perturbations with a flat spectrum that are the origin of the inhomogeneities of matter in the present universe. However, the simplest GUT predicts the value of density perturbations with a magnitude  $10^5$  times larger than what is observed (Guth, 1992): indeed some theories lead to too large density fluctuations after inflation and therefore have been rejected. On the contrary, in some other theories the amplitude of density perturbations has the magnitude which is just necessary for galaxy formation and compatible with the limits of COBE on the anisotropies of the CMBR.

Unfortunately, the inflation paradigm does not solve the superluminal expansion and the inhomogeneity paradoxes. Moreover, it rather magnifies superluminal expansion. For example, if a distance between two particles before inflation was  $l_0 = 10^{-35} cm$ , after the inflationary era it will be  $10^{4 \cdot 10^8} cm$ , hence the velocity of expansion will be (Novikov, 1988):

$$v \sim l_0 / \Delta t \sim 10^{4 \cdot 10^8} cm/sec \gg c \quad (3.82)$$

There are two explanations of the relation in Eq.3.82. The first is that there is nothing in General Relativity that constraints the speed of space expansion (see Guth, 1992). The second is that there is no possibility to measure the relative velocity of distant objects in a strong gravitational field, i.e. there is no sense in the notion of the relative velocity for the two particles which was considered above (Novikov, 1988). However, if there is no possibility to measure the relative velocity of distant particles, then one can not measure the distance between the particles too, because one has no rigid measuring rod. Hence there is no such concept as distance in inflating Universe. So the inflation paradigm uses the highly superluminal expansion of space to carry information about local physical conditions for almost the whole Universe. Hence superluminal expansion paradox is still unsolved within inflation models.

Recently a modified version of the Inflationary scenario has been introduced by Linde (1990, 1994). In this model, due to quantum fluctuations of the scalar fields, the universe exists eternally

as a huge self-reproducing entity and it is extremely inhomogeneous, with a fractal like structure, on scales larger than the casually connected horizon. This fractal structure has nothing to do with the fractal distribution of matter in the visible Universe, but it concerns the distribution of many universes that undergo, at different times, an inflationary process.

According to Linde (1994) there are some testable predictions of the inflationary model. First of all the inflation theories predict that the Universe is extremely flat with an average density equal to the critical one. A crucial point of the inflationary scenario is that the density has to be very close to the critical one: this is connected with the famous dark matter problem that is still unsolved. Another testable prediction is related to density perturbations produced during inflation that, in some models, are in agreement with the results of COBE. The scalar fields themselves are not directly observable quantities, if they fill homogeneously the Universe but their presence affects the properties of elementary particles. In the GUT of weak and strong interactions, there are two scalar fields and it would be necessary to verify experimentally the GUT theories. Unfortunately, this appears to be a very difficult task as the energy scale is about that of the Planck mass rest energy  $\sim 10^{19} \text{Gev}$ , while the currently available particle accelerators reach  $\sim 10^4 \text{Gev}$ . There are some indirect tests like the proton decay and the search for supersymmetric particles, but there are no definite results. In any case there is not a well defined candidate for the inflation scalar field.

### 3.3 Steady-State Models

The Steady State Model (hereafter SSM) was proposed by Bondi and Gold (1948) and Hoyle (1948) as a competing alternative to the Standard Hot Big Bang Model. More recent reviews of the SSM can be found in Hoyle (1982, 1991), Narlikar (1977, 1987, 1993) and Hoyle et al. (1993, 1994a, 1994b).

#### 3.3.1 Basic Hypotheses of the SSM

The *first hypothesis* of SSM is the Perfect Cosmological Principle (PCP), i.e. globally unchanging Universe in space and time. According to PCP there is no global evolution of the Universe so that the mean matter density does not change with time. Bondi and Gold emphasize that the main reason for PCP is the conservation of the cosmological external conditions which allow us to use terrestrial physics unambiguously in cosmology. The *second hypothesis* is that gravitation theory is a modified version of General Relativity, i.e. it is a geometrical description of gravity. There are several versions of the gravitational field equation within the SSM. In Hoyle-Narlikar's approach a scale invariant gravitation theory is used so that the creation of matter by means of the *C-field* is possible. It does not violate the law of energy conservation because the theory is derived from an action principle and therefore, because of Noether's theorem, it obeys the conservation laws. The *C-field* has negative energy and negative stresses and it is conserved together with usual matter. The *third hypothesis* is that the observed cosmological redshift is caused by the space expansion, i.e. increasing space volume with time ("creation of space"). To compensate the decreasing matter density it is assumed the creation of matter ("creation of space with matter") so the PCP is fulfilled.

#### 3.3.2 Prediction for testing

In the SSM the Universe had no beginning and so it is infinitely old, the density of the Universe remains constant, the matter and space are being continuously created, and the Universe is not evolving. The Robertson-Walker line element in the SSM is characterized by the following parameters: 1) the space curvature  $k = 0$ ; 2) the Hubble constant  $H$  is a fundamental constant; 3) the

deceleration parameter  $q = -1$ . Hence the scale factor is :

$$R(t) = Ae^{Ht} \quad (3.83)$$

where  $A$  is constant, and the density of the Universe has the critical value:

$$\rho = \frac{3H^2}{8\pi G} \quad (3.84)$$

For metric distance we get

$$l = R_H z \quad (3.85)$$

where  $R_H = c/H$ . Hence the angular size-redshift relation has the form

$$\Theta(z) = \frac{d}{R_H} \frac{(1+z)}{z} \quad (3.86)$$

the bolometric magnitude-redshift relation is:

$$m_{bol}(z) = 5 \log(z(1+z)) + const. \quad (3.87)$$

and the integral count-redshift is

$$N(z) = const \cdot \left( \log(1+z) - \frac{z(2+3z)}{2(1+z)^2} \right) \quad (3.88)$$

It is clear that there are no flatness and isotropy paradoxes in the SSM. However the superluminal expansion, inhomogeneity and global energy paradoxes still exist in the SSM. The main observational difficulty that the SSM faces, is, however, the CMBR: an adequate theory that should produce the correct spectrum and isotropy has not been developed yet.

But, as it was mentioned by Hoyle (1991), a small fractional conversion of baryonic energy into electromagnetic energy (e.g. by stellar nuclear reactions) inevitably yields the correct order of magnitude for the energy of microwave background. The problem consists of the thermalization of this energy up to its thermodynamic value  $2.73K$ .

### 3.3.3 Modern development of the SSM

An interesting argument in favor of the SSM has been found by Hoyle and Narlikar (1963) (see also Hoyle & Narlikar, 1974; Narlikar, 1977). They considered the Wheeler-Feynman absorber theory in the context of various cosmological models to see which of them give the correct response of the Universe. We can divide the rest of the Universe with respect to the present position of the electric charge  $A$  into two light cones in the future and the past. The cone and its interior in the future is the "future absorber", while the cone in the past is the "past absorber". The rule for the correct response from a Universe model is that the past absorber is imperfect and the future absorber is perfect (it must totally absorb all disturbances generated by  $A$ ).

In the static Universe without redshift both absorber are perfect and the result is ambiguous, i.e. one can get retarded or advanced waves. In the open Friedmann models the density of the matter in the future absorber decreases to zero and there is not enough matter to absorb radiation. Thus the future absorber is imperfect and we get advanced radiation. In the closed Friedmann models the result is ambiguous as in the Static Universe. SSM only has the correct response of the Universe and hence predicts the retarded radiation.

Recently Hoyle, Burbidge and Narlikar (1993; 1994a; 1994b) have modified the SSM. In the new "Quasi Steady State" model there are scale invariant gravitational equations, which

reduce to those of General Relativity for a particular choice of the scale. The authors have shown that the model can explain the  $2.73K$  CMBR and the abundances of the light elements  $D$ ,<sup>3</sup>  $He$ ,<sup>4</sup>  $He$ ,<sup>6</sup>  $Li$ ,<sup>7</sup>  $Li$ ,<sup>9</sup>  $Be$ ,<sup>11</sup>  $B$ . The model is based on the idea of discrete creation events. These creation events occur throughout the whole Universe, in creation centers. The most important creation centers have masses of the order  $10^{16}M_{\odot}$ . The newly created particles have Planck mass  $(hc/2\pi G)^{1/2}$  and are unstable over Planck time scale  $(hG/2\pi c^5)^{1/2}$ . The created masses are much less than  $10^{16}M_{\odot}$ , related to such well-known events as young galaxies, active galactic nuclei, galactic groups and clusters with positive total energy. Within the model there was a major creation episode when the mean universal density was  $10^{-27}gr \cdot cm^{-3}$ . Since then the universal expansion has been slowing down with the parameter  $q_0$  rising from -1 to its present day value  $q_0 = +1$ .

### 3.4 Fractal models

At present a fractal model has not yet been developed, but there are some attempts and many possibilities are still under consideration. For this reason we firstly illustrate the main properties of self-similar structures, showing why a change of perspective is required if one deals with fractals. In fact, self-similar structures are intrinsically irregular (non-analytic) at all scales and fractal geometry allows us to characterize them mathematically. There are some deep implications of these concepts not only on the theoretical side but also on the data analysis. The main theoretical consequence is that one should not discuss a self-similar structure in terms of amplitudes of correlation, but the only meaningful quantity is the exponent that characterizes the power law behaviour. We discuss in detail the observational properties of large-scale structure distribution that one can obtain from the available redshift surveys with an appropriate statistical analysis.

From a theoretical point of view the dark matter plays a crucial role, because if it is homogeneously distributed the Friedmann metric can still be the right one, even if a deep revision of the models of the large-scale structure formation is required. On the contrary, if the dark matter turns out to be distributed as the visible one, the problem becomes very hard and has to be reconsidered from the beginning. The test on the whole matter distribution, provided by the analysis of bulk flows and gravitational lensing effects, is therefore a crucial one.

We briefly illustrate the old idea of hierarchical cosmology, based on the Newtonian theory of gravitation, that has behind it the same concept of scale-invariance of fractal structures. Hence we consider some attempts to reconcile the fractal distribution in the framework of General Relativity, in spherically symmetric models described by the Tolman-Bondi metric. Finally we discuss a new idea based on tensor field relativistic gravitation theory in Minkowski space-time.

#### 3.4.1 Fractal geometry

Fractals are simple but subtle. In this section we provide a brief description of their essential properties. This description is not intended to represent a theory, but only to illustrate the consequences of the properties of self-similarity so that, if the property is actually present in the experimental data, we will be able to detect it correctly; on the contrary, if the data would be in contrast with the fractal properties, we have to know well the properties of fractals in order to eventually conclude that observations are actually in contrast with them. Fractal geometry has a long history and some elements of it can be found already in the work of Poincaré and Hausdorff of about a century ago. The introduction of the name "*Fractals*" and the realization that fractal geometry is a powerful tool to characterize intrinsically irregular systems is due to Mandelbrot and it refers mainly to the past twenty years (Mandelbrot, 1982). Nature is full of strongly irregular structures; trees, clouds, mountains and lightning are quite familiar objects but are very different

from the structures of euclidean geometry. A common element of these and many other structures is that if one magnifies a small portion of them, this reveals a complexity comparable to that of the entire structure. This is geometric self-similarity and it has deep implications for the non-analyticity of these structures. In fact analyticity or regularities would imply that at some small scale the profile becomes smooth and one can define a unique tangent. Clearly this is impossible in a self-similar structure because at any small scale a new structure appears and the structure is never smooth. Self-similar structures are therefore intrinsically irregular at all scales and this is why many familiar phenomena have remained at the margins of scientific investigation.

The usual mathematical concepts in physics are mostly based on analytical functions and, in this perspective, irregularities are seen as imperfections. Fractal geometry changes completely this perspective by focusing exactly on these intrinsic irregularities and it allows us to characterize them in a quantitative mathematical way. In this way it has extended the frontiers of scientific investigation to all the intrinsically irregular structures. It should be made clear however that fractal geometry is not a physical theory because it does not explain why nature generates fractal structures. It permits us however to look at irregularities in a new way and to pose the correct questions for a theory. Fractal geometry has had a large impact in all sciences and in particular in physics. In fact in physics the concept of scale invariance was already familiar from the study of critical phenomena in phase transitions. This is an example of self-similarity that can be understood with the Renormalization Group theory. This method however does not seem to be very effective for the more familiar fractal structure that arises from irreversible dynamics and this is the main theoretical challenge at the moments. Fractal concepts have become very popular and even fashionable in many areas for various reasons. The characterization of intrinsic irregularities represents a new powerful tool to understand the properties of nature. Also the aesthetic appeal of these structures and the return of visual intuition into science have played an important role.

In astrophysics however these ideas have been considered with skepticism and sometimes even with opposition. The main reason is that they may appear to contrast with concepts like the Cosmological Principle and the whole standard theoretical framework based on homogeneity and the Friedmann metric. On the other hand the large-scale structures observed experimentally provide a strong evidence for self-similar and fractal properties. So we have to hope that after the first period of emotional debate the situation can evolve into constructive confrontation.

### 3.4.2 Fractal Structures

A fractal consists of a system in which more and more structures appear at smaller and smaller scales and the structures at small scales are similar to the one at large scales. In Fig.2(a), we show an elementary (deterministic) fractal distribution of points in space whose construction is self-evident. Starting from a point occupied by an object we count how many objects are present within a volume characterized by a certain length scale in order to establish a generalized "mass-length" relation from which one can define the fractal dimension.

Suppose that in the structure of Fig.2(a) we can find  $n$  objects in a volume of size  $r_0$ . If we consider a larger volume of size  $r_1 = k \cdot r_0$  we will find  $N_1 = \tilde{k} \cdot N_0$  objects. In a self-similar structure the parameters  $k$  and  $\tilde{k}$  will be the same also for other changes of scale. So, in general in a structure of size  $r_n = k^n \cdot r_0$  we will have  $N_n = \tilde{k}^n \cdot N_0$  objects. We can then write a relation between  $N$  ("mass") and  $r$  ("length") of type:

$$N(r) = B \cdot r^D \quad (3.89)$$

where the fractal dimension:

$$D = \frac{\log \tilde{k}}{\log k} \quad (3.90)$$

Figure 2: (a) A simple example of a deterministic fractal with dimension  $D=1.2$ . The same structures repeats at different scales in a self-similar way. (b) Example of a stochastic fractal with dimension  $D= 1.2$  generated by the "random  $\beta$ -model".

depends on the rescaling factors  $k$  and  $\tilde{k}$ . The prefactor  $B$  is instead related to the lower cut-offs  $N_0$  and  $r_0$ ,

$$B = \frac{N_0}{r_0^D} \quad (3.91)$$

It should be noted that Eq.3.89 corresponds to a smooth convolution of a strongly fluctuating function as evident from Fig.2. Therefore a fractal structure is always connected with large fluctuations and clustering at all scales. From Eq.3.89 we can readily compute the average density  $\langle n \rangle$  for a sample of radius  $R_s$  which contains a portion of the fractal structure. The sample volume is assumed to be a sphere ( $V(R_s) = (4/3)\pi R_s^3$ ) and therefore

$$\langle n \rangle = \frac{N(R_s)}{V(R_s)} = \frac{3}{4\pi} B R_s^{-(3-D)} \quad (3.92)$$

From Eq.3.92 we see that the average density is not a meaningful concept in a fractal because it depends explicitly on the sample size  $R_s$ . We can also see that for  $R_s \rightarrow \infty$  the average density  $\langle n \rangle \rightarrow 0$ , therefore a fractal structure is asymptotically dominated by voids. We see therefore that the average density  $\langle n \rangle$  is not a well defined quantity but the *conditional average* density as given by Eq.3.93 is well defined in terms of its exponent, the fractal dimension. The amplitudes of these functions essentially refer to the unit of measures given by the lower cut-offs but they have no particular physical meaning because they are not intrinsic quantities. We can also define the conditional density from any occupied point as:

$$\Gamma(r) = S^{-1} \frac{dN(r)}{dr} = \frac{D}{4\pi} B r^{-(3-D)} \quad (3.93)$$

where  $S(r)$  is the area of a spherical shell of radius  $r$ . Usually the exponent that defines the decay of the conditional density  $(3-D)$  is called the codimension and it corresponds to the exponent  $\gamma$  of the galaxy distribution. In Fig.2(b) we show a stochastic fractal (generated with the random- $\beta$ -model algorithm in the two dimensional euclidean space) constructed with a probabilistic algorithm that nevertheless has a well defined fractal dimension  $D = 1.2$ .



### 3.4.2.1. Mathematical Self-similarity

From Fig.2(a) the geometrical self-similarity is evident in the construction. From a mathematical point of view self-similarity implies that a rescaling of the length by a factor  $b$

$$r \rightarrow r' = br \quad (3.94)$$

leaves the correlation function unchanged apart from a rescaling that depends on  $b$  but not on the variable  $r$ . This leads to the functional relation

$$\Gamma(r') = \Gamma(b \cdot r) = A(b) \cdot \Gamma(r) \quad (3.95)$$

which is clearly satisfied by a power law with any exponent. In fact for

$$\Gamma(r) = \Gamma_0 r^\alpha \quad (3.96)$$

we have

$$\Gamma(r') = \Gamma_0 (br)^\alpha = (b)^\alpha \Gamma(r) \quad (3.97)$$

The same does not hold, for example, for an exponential behavior

$$\Gamma(r) = \Gamma_0 e^{-r/r_0} \quad (3.98)$$

This reflects the fact that power laws do not possess a characteristic length while for the exponential decay  $r_0$  is a characteristic length. Note that the characteristic length has nothing to do with the prefactor of the exponential and it is not defined by the condition  $\Gamma(r_0) = 1$ , but from the intrinsic behavior of the function. This brings us to a common misconception that sometimes occurs in the discussion of galaxy correlations. Even for a perfect power law as Eq.3.96 one might use the condition  $\Gamma(r_0) = 1$  to derive a "characteristic length":

$$r_0 = \Gamma_0^{-1/\alpha} \quad (3.99)$$

This however is completely meaningless because the power law refers to a fractal structure constructed as self-similar and therefore without a characteristic length. In Eq.3.99 the value of  $r_0$  is just related to the amplitude of the power law that, as we have discussed, has no physical meaning. The point is that the value 1 used in the relation  $\Gamma(r_0) = 1$  is not particular in any way so one could have used as well the condition  $\Gamma(r_0) = 10^{10}$  or  $\Gamma(r_0) = 10^{-10}$  to obtain other lengths. This is the subtle point of self-similarity; there is no reference value (like the average density) with respect to which one can define what is big or small.

### 3.4.2.2. "Linear and Non-linear Dynamics"

Such a discussion naturally extends to the much discussed value of  $\delta N/N$  the ratio of density fluctuations with respect to the average density. Another argument often mentioned in the discussion of large-scale structures is that it is true that larger samples show larger structures but their amplitudes are smaller and the value of  $\delta N/N$  tends to zero at the limits of the sample; therefore one expects that just going a bit further, homogeneity would finally be observed. Apart from the fact that this expectation has been systematically disproved, the argument is conceptually wrong for the same reasons of the previous discussion. In fact we can consider a portion of a fractal structure of size  $R_s$  and study the behavior of  $\delta N/N$ . The average density  $N$  is just given by Eq.3.92 while the overdensity  $\delta N$ , as a function of the size  $r$  of a given in structure is ( $r \leq R_s$ ):

$$\delta N = \frac{N(r)}{V(r)} - \langle n \rangle = \frac{3}{4\pi} B(r^{-(3-D)} - R_s^{-(3-D)}) \quad (3.100)$$

Figure 3: Behavior of  $\delta N/N$  as a function of the size  $r$  in a portion of a fractal structure for various depths of the sample:  $R_s = 100, 200, 300 Mpc$ . The average density is computed over the whole sample of radius  $R_s$ . The fact that  $\delta N/N$  tends to zero does not mean that the fluctuations are small and a homogenous distribution has been reached. The scale at which  $\delta N/N = 1$  scales with sample depth and has no physical meaning. In the case of a fractal distribution the normalization factor, i.e. the average density, is not an intrinsic quantity.

We have therefore

$$\frac{\delta N}{N} = \left( \frac{r}{R_s} \right)^{-(3-D)} - 1 \quad (3.101)$$

Clearly for structures that approach the size of the small sample, the value of  $\delta N/N$  becomes very small and eventually becomes zero at  $r = R_s$  as shown in Fig.3. This behavior however could be interpreted as a tendency towards homogeneity because again the exercise refers to a self-similar fractal by construction. Also in this case the problems come from the fact that one defines an "amplitude" arbitrary by normalizing with the average density that is not an intrinsic quantity. A clarification of this point is very important because the argument that since  $\delta N/N$  becomes smaller at large scale, there is a clear evidence of homogenization is still quite popular (Peebles et al., 1991; Peebles, 1993; Peebles et al., 1994) and it adds confusion to the discussion.

The correct interpretation of  $\delta N/N$  is also fundamental for the development of the appropriate theoretical concepts. For example a popular point of view is to say that  $\delta N/N$  is large ( $\gg 1$ ) for small structure and this implies that a non linear theory will be necessary to explain this. On the other hand  $\delta N/N$  becomes small ( $< 1$ ) for large structures, which require therefore a linear theory. The value of  $\delta N/N$  has therefore generated a conceptual distinction between small structures that would entail non linear dynamics and large structures with small amplitudes that correspond instead to linear dynamics. If one would apply the same reasoning to our example of a fractal structure we would conclude that for a structure up to a size (from Eq.3.101):

$$r^* = 2^{-\left(\frac{1}{3-D}\right)} R_s \quad (3.102)$$

we have  $\delta N/N$  so that a non linear theory is needed. On the other hand for large structures ( $r > r^*$ ) we have  $\delta N/N$  corresponding to a linear dynamics. Since the fractal structure that we have used to make this conceptual exercise has scale invariant structures by construction, we can see the distinction between linear and non linear dynamics is completely artificial and wrong. The point is again that the value of  $N$  we use to normalize the fluctuations is not intrinsic but it just reflects the size of the sample that we consider ( $R_s$ ).

If we have a sample with depth  $\tilde{R}_s$  greater than the eventual scale of homogeneity  $\lambda_0$ , then the average density will be constant in the range  $\lambda_0 < r < \tilde{R}_s$  apart from small amplitude fluctuations. The distance at which  $\delta N/N = 1$  will be given by:

$$r^* = 2^{-(\frac{1}{3-D})} \lambda_0 \quad (3.103)$$

If, for example,  $D = 2$  and  $\lambda_0 = 200 Mpc$  then  $r^* = 100 Mpc$ : therefore a homogeneity scale of this order of magnitude is incompatible with the standard normalization of  $\delta N/N = 1$  at  $8h^{-1} Mpc$ .

We can see therefore that the whole discussion about large and small amplitudes and the corresponding non linear and linear dynamics has no meaning until an unambiguous value of the average density has been defined, so that the concepts like large and small amplitudes can take a physical meaning and be independent of the size of the catalogue. It is also an instructive exercise to consider the behavior of the function

$$\xi(r) = \frac{< n(r_0)n(r_0+r) >}{< n >^2} - 1 \quad (3.104)$$

for a fractal structure. The result is:

$$\xi(r) = \left( \frac{3-\gamma}{3} \right) \left( \frac{r}{R_s} \right)^{-\gamma} - 1 \quad (3.105)$$

with  $\gamma = 3 - D$ . In this case also if one defines a length by the relation  $\xi(r_0) = 1$ , one obtains:

$$r_0 = \left( \frac{3-\gamma}{6} \right)^{\frac{1}{\gamma}} R_s \quad (3.106)$$

that has no physical meaning because it is dependent on the sample size  $R_s$  and not on the real correlations of the system. The basic point of all this discussion is that in a self-similar structure one cannot say that correlations are "large" or "small", because these words have no physical meaning due to the lack of a characteristic quantity with respect to which one can normalize these properties. The deep implication of this fact is that one cannot discuss a self-similar structure in terms of amplitudes of correlation. The only meaningful physical quantity is the exponent that characterizes the power law behaviour. Note that this problem of the "amplitude" is not only present in the data analysis but also in the theoretical models. Meaningful amplitudes can only be defined once one has unambiguous evidence for homogeneity but this is clearly not the case for galaxy and cluster distributions.

In this section we have seen with a few examples what type of problems and misleading conclusions one may derive from statistical analysis performed in a naive way. Of course the example of a fractal structure that we have used, is just for conceptual clarification and it should not be confused with a theory. We hope however that this discussion will help to clarify misconceptions like that if  $\delta N/N$  becomes small at large scales this is an evidence for homogenization with all the related consequences and theoretical implications.

It is therefore clear that a crucial element to consider in the analysis of galaxy and cluster distributions is whether a real homogenization is achieved and average density can be derived. Only in this case in fact do the amplitudes of correlation acquire a physical meaning and one can consider it as a theoretical problem. In the opposite case and anyhow for the range of scales in which the structure is self-similar (even if homogeneity is eventually achieved at large scale) it is necessary to change the theoretical language and perspective and adopt the one that is appropriate for self-similar and non-analytical structures.

### 3.4.2.3. Fair Sample

In this perspective it is important to clarify the concept of "Fair Sample". Often this concept is used as synonymous of a homogeneous sample (see for example da Costa et al., 1994). So the analysis of catalogues along the traditional lines often leads to the conclusion that we still do not have a fair sample and deeper surveys are needed to derive the correct correlation properties. A corollary of this point of view is that since we do not have a fair sample its statistical analysis cannot be taken too seriously.

This point of view is highly misleading because we have seen that self-similar structures never become homogeneous, so any sample containing a self-similar (fractal) structure would automatically be declared "not fair" and therefore impossible to analyze. The situation is actually much more interesting otherwise the statistical mechanics of complex systems would not exist. Homogeneity is a property and not a condition of statistical validity of the sample. A non homogeneous system can have well defined statistical properties in terms of scale invariant correlations, that may be perfectly well defined. The whole studies of fractal structures are about this (Pietronero & Tosatti, 1986). Therefore one should distinguish between a "statistical fair sample" which is a sample in which there are enough points to derive some statistical properties unambiguously and a homogeneous sample that is a property that can be present or not but that has nothing to do with the statistical validity of the sample. In the following we are going to see that even the small sample like *CfA 1* is statistically fair up to a distance that can be defined unambiguously. Also the combined sample *CfA2* and *SSRS2* has been declared to be "*not fair*" (da Costa et al., 1994).

#### 3.4.2.4. *Isotropy, Homogeneity and Cosmological Principle*

The Cosmological Principle (CP) implies that all the mass points (galaxies) should be statistically equivalent with respect to their environment. This condition corresponds to the property of local isotropy and it is generally believed that the universe cannot be isotropic about every point without being also homogeneous. Actually local isotropy does not necessarily imply homogeneity. In fact a topology theorem states that homogeneity is implied by the condition of local isotropy together with the assumption of analyticity or regularity for the distribution of matter (Weinberg 1972). It is easy to see (Fig.2) and to prove that in fractal structure the condition of local isotropy, or statistical equivalence of all occupied points, is actually satisfied but, since the structure is non-analytical, the property of homogeneity is not implied. This means that a fractal structure is in perfect agreement with the CP. In addition this also implies that the tests of dipole moment saturation are only tests of local isotropy, but not homogeneity. We have shown in fact that the dipole moment saturates quickly also in a fractal structure while the monopole moment grows as the power law of the sample size (Sylos Labini, 1994).

#### 3.4.2.5. *Power Spectrum*

The power spectrum (PS) measures the fluctuation amplitude of the density field as a function of scale. The amplitudes of the Fourier modes with different wavenumber  $\vec{k}$  are uncorrelated: the phases carry all information about higher moments of the spatial distribution. The main problem of the PS of the normalized density fluctuations is that it is not suitable to describe a highly irregular systems and does not measure the cut-off towards homogeneity, but it is useful to characterize a system with a well defined average density and small fluctuation around it.

For fractal distribution Sylos Labini et al.(1995) found that the amplitude of the PS is a power law function of the sample depth so that it has no physical meaning but it is only the reflex of the sample size. Moreover the shape of the PS is characterized by two scaling regimes: the first one, at high wavenumbers, is related to the fractal dimension of the distribution in real space. The second one, that can be seen as a flattening of the PS for  $k \rightarrow 0$ , arises because of a finite size effect and it is spurious. See for example Park et al. (1994) for an analysis of the PS of real data.

In order to perform an analysis that does not imply any a priori assumption one should study the PS of the density not normalized to the average, that is characterized, in the case of fractal distributions, by a single power law behaviour; moreover the amplitude does not depend on the sample size.

### 3.4.3 Fractal Properties of visible matter

At this point we have all the elements to perform a correlation analysis for the galaxy and cluster distribution. Of course this has been performed extensively in the past using the function  $\xi(r)$  (Eq.3.104) and related concepts. As we have seen, this approach can be misleading if the distribution is not really homogeneous within the sample limits. Since this is clearly not the case, the usual analysis of correlations is considered problematic. This situation has induced various authors to consider alternative methods of analysis like the power spectrum, topological methods, void distribution, percolation analysis and even more.

However, given the distribution of the previous section, we are now in the position to understand what is the real reason for the ambiguity of the analysis with  $\xi(r)$  as well as the problem of "fair" samples versus "homogeneous" samples. In our opinion therefore, the most important statistical information that we should derive from the catalogues is still the fair correlation properties. In particular, there is general agreement that, at relatively small scales, correlations show a power law behaviour consistent with clustering and fractal properties. The crucial question is, therefore, about the eventual tendency or evidence for homogenization. Since this is the property we want to test, it is not wise to assume it "a priori", as it is usually done in the  $\xi(r)$  analysis, but also in the various "weighting schemes". In practice the analysis is quite simple: one should consider the conditional density or the average conditional density and check whether the power law behaviour (*fractal*) at small scales is followed by a well defined constant behavior over appreciable distances, within the limits of statistical validity of the sample.

#### 3.4.3.1. Galaxies and Clusters

For the *CfA 1* redshift survey (Huchra et al., 1983) we performed this analysis in 1988 (Coleman et al., 1988) and the result was quite surprising:

i.) the limit of statistical validity of this sample was found to be

$$R_G \sim 20Mpc \quad (3.107)$$

that is about the radius of the maximum sphere that can be contained in the sample. This means that, contrary to the common opinion, we are in the presence of a statistically "fair sample" up to this distance.

ii.) The analysis of the appropriate correlation function shows a well defined power law behavior up to  $R_s$ , without any tendency towards homogenization (Fig.4). This means that *CfA 1* essentially contains a portion of a fractal structure with dimension

$$D \sim 1.4 \quad (3.108)$$

This means that all the warnings about possible inconsistency of the  $\xi(r)$  analysis are actually justified and that, the well known "correlation length"  $r_0 \sim 5Mpc$  (Davis & Peebles, 1983), is a spurious result that reflects just the size of the sample and not the real correlation properties of the system.

The same type of analysis was later (Coleman & Pietronero, 1992) performed also for the *Abell* catalogue of clusters (Abell, 1958; Bahcall, 1988). Also in this case we have observed fractal correlation up to the effective size of the sample ( $R_s \sim 80Mpc$ ), with a value of the fractal

Figure 4:  $\Gamma(r)$ ,  $\Gamma^*(r)$  and  $\xi(r)$  plotted as a function of the length scale for the *CfA1* catalog limited to  $v \leq 8000 \text{ km sec}^{-1}$ . The dashed line indicates a reference slope of 1.7. The CfA does not show any real correlation length within the limit of its statistical validity but it shows well defined power law correlations (from Coleman & Pietronero, 1992).

dimension similar to that of galaxies (Fig.6). This leads us now to an important point that shows, in a very concrete way, how, looking at the same experimental data with a broader conceptual scheme, leads to a physical problematic of a completely different nature. We have seen in the previous section that for fractal structures the amplitude of correlations is physically meaningless, because it reflects just the unit of measures and the artificial cut-off, corresponding to the size of the considered sample. Then we have seen the *CfA 1* catalogue of galaxies just contains a portion of fractal distributions. This means that galaxies are clustered in a self-similar way. By taking as units some of these clusters, one obtains the *Abell* catalogue of clusters for which we also have a fractal distribution with about the same fractal dimension. Since clusters are made by galaxies, it is then quite natural to consider that cluster correlations are just the continuation of galaxy correlations at larger scales. This would imply that the correlations lengths (derived from correlation amplitudes)  $r_0^G \sim 5 \text{ Mpc}$  for galaxies and  $r_0^C \sim 25 \text{ Mpc}$  for clusters are just a finite fraction of the depth of the galaxy  $R_G$  and cluster  $R_C$  catalogues. We find (Coleman & Pietronero, 1992) in fact that

$$\frac{r_0^C}{r_0^G} \sim \frac{R_C}{R_G} \sim 5 \quad (3.109)$$

that provides, together with the coincidences of the exponents, a strong evidence for the fact that galaxy and cluster catalogues correspond to a single fractal distribution of galaxies, that is looked at from different points of view in the two catalogues (Fig.5). Therefore, the famous problem of the galaxy cluster mismatch (Bahcall, 1986) was just due to the inappropriate statistical analysis and it is automatically eliminated by a correct statistical description.

This work has generated a debate and it may be worth to report some of its elements. For example, some authors claim (Lemson & Sanders, 1992; Provenzale et al., 1994) that the use of weighting schemes allows us to extend the limits of statistical validity of the samples and, by doing so, they claim to observe trends towards homogenization outside our limits for both galaxy and cluster catalogues. Our opinion, about this point, is that any weighting schemes must be based on a model and the basic idea is to replicate what is missing in the sample, using the knowledge of what is available. Unavoidably, this introduces artificial homogenization effects so, it is not a

Figure 5: Ideal catalogue of galaxies and clusters. The correlation of clusters appears to be the continuation of the galaxy correlation to larger scales: the points are galaxies and the groups of galaxies are clusters. The depth of clusters catalogue is deeper than that of the galaxy catalogues because clusters are brighter than galaxies.

Figure 6:  $\Gamma(r)$ ,  $\Gamma^*(r)$  and  $\xi(r)$  plotted as a function of the length scale for the Abell clusters catalog. As the *CfA1* galaxies catalog the sample shows well defined power law correlations and no correlation length (from Coleman & Pietronero, 1992).

surprise that the claimed scales of homogenization are about  $35Mpc$  for galaxies and  $150Mpc$  for clusters. Both values are just about twice the limits of statistical validity of these catalogues and, therefore, they refer to length scales that are strongly affected by the weighting schemes. In addition to these comments, we are going to see that deeper catalogues clearly contradict these claims for homogeneity.

From a theoretical point of view the mismatch between galaxy and cluster correlations was interpreted in many ways. One is the so called *biased galaxy formation* model (Kaiser, 1984). This model starts from a reasonable idea, namely a galaxy grows by taking matter around it. The point is that its present mathematical description, in terms of an uncorrelated random process (Poisson), does not reproduce even the qualitative features of the power law behaviour of galaxy and cluster correlations. However an argument is made about the amplitude of correlation between small and large fluctuations of a Poisson process and it is then related to the fictitious amplitude mismatch of power law correlations. A related concept, that was also generated by these fictitious amplitudes, is the called "luminous segregation effect". This was actually not even defined theoretically and it is in fact impossible to generate this effect as a continuous distribution of masses. However it appeared as the continuum generalization of the galaxy cluster mismatch. This confused situation can be now clarified naturally on the basis of the previous discussion and the objective of a theory can finally be identified in a clear and consistent way.

### 3.4.3.2. The CfA 2 redshift survey

Recently an extension of the *CfA 1* catalogues up to a magnitude limit of 15.5, corresponding to a depth of about twice, has become available for an appreciable angular volume. This survey confirms and extends our previous results for *CfA 1*. The power law (fractal) correlations observed up to the sample limit of the *CfA 1* are present also up to the depth of *CfA 2* (Park et al., 1994; da Costa et al., 1994). This implies that, also in the extended *CfA 2* catalogue, there is no tendency towards homogenization (Pietronero & Sylos Labini, 1995). Unfortunately, this clear trend in the correlation properties is misinterpreted with the statement that, even *CfA 2*, is not a "fair sample" (Da Costa et al. 1994), simply because it is not homogeneous. The exponent of the correlation function is lower than in *CfA 1* and it is  $\gamma \sim 1.1$ , so that the fractal dimension is  $D = 3 - \gamma \sim 1.9$ , somewhat higher than in *CfA 1*. The amplitude of the correlation function  $\xi(r)$  is a linear function of the sample depth up to  $130Mpc$  (Park et al., 1994). From the comparison of the volume limited subsamples of *CfA 2* with those of *CfA 1* with the same limiting absolute magnitude, it follows that, the linear growth of the correlation function amplitude (or the power spectrum amplitude) in deeper subsamples cannot be due to a luminosity bias otherwise it should be constant in those subsamples. On the contrary, the observed linear dependence of  $r_0$  with the sample depth can be naturally explained by the fractal nature of the galaxy distribution in the *CfA2* survey (Pietronero & Sylos Labini, 1995). The availability of the *CfA2* catalogue brings us to another point that has been until now used in favor of homogeneity. Some authors (Peebles, 1993) in fact argued that, a certain rescaling of the angular correlations would actually imply homogeneity. This argument is based on functions that are the analogues of  $\xi(r)$  for angular correlations, so it suffers from the problems we have previously discussed with the additional one that the angular correlation corresponds to complex projections, that lead to further complications. In any case, one of the three angular catalogues used in the discussion of the angular correlations is the Zwicky catalogue, that corresponds to the angular properties of the *CfA2* catalogue. Now we have the full three dimensional distribution for the catalogue and we can clearly see that nothing happens in the correlations at  $5Mpc$  and that no tendency towards homogeneity is present in the whole sample.

Concerning the fact that the angular catalogues are really smoother than the three dimensional ones, this is due to the complex properties of angular projections that, contrary to orthogonal projections, mix different length scales and produce an artificial effect of homogenization. For



Figure 7: Wedge diagram of the *ESP* survey (from Vettolani et al., 1994).

example, we have shown (Dogterom & Pietronero, 1991; Coleman & Pietronero, 1992) that the angular projection of a fractal structure becomes really homogenous at large angular scales. This shows that a smooth angular projection does not imply the same property in the real distribution.

#### 3.4.3.3. *Pencil beams and very deep surveys*

In the past few years there has been a large interest in tiny but very deep samples (pencil beams). These samples extend over length scales of the order of  $1000Mpc$ , covering a small solid angle in the sky: they show a strong irregularity of the distribution of galaxies over their entire length (Broadhurst et al., 1990; Kirshner et al., 1978). This is, again, a confirmation of the absence of homogeneity up to very large distances. Curiously however, the attention has been focused mainly on the apparent periodicity of the peaks in the galaxy density. Apart from the fact that the peak intensities are extremely different, the period is very large ( $\sim 130Mpc$ ) so that the eventual homogeneity, corresponding to this periodicity, could be achieved above  $1000Mpc$ . In addition, new surveys of this type show less and less evidence for the periodicity, while they confirm the irregularities (for a more detailed discussion of pencil beams data see also Coleman & Pietronero, 1992).

The accumulation of many pencil beam surveys has produced the Eso Slice Project (hereafter *ESP*) (Vettolani et al., 1994), that we have recently analyzed. We find that, analyzing the number-redshift relation, this sample does not seem to show any tendency towards homogenization up to its depth ( $600 - 800Mpc$ ) (Pietronero & Sylos Labini, 1994; Sylos Labini & Pietronero, 1995a). The distribution of galaxies is fractal with dimension  $D \approx 2$  up to the sample limits. This value is larger than the value previously found for *CfA1*, but it is in agreement with various other deeper redshift surveys such as *CfA2* (Park et al., 1994), *Peruses-Pisces* (Guzzo et al., 1992) and *QDOT* (Moore et al., 1994).

#### 3.4.4 Multifractal properties of luminous matter distribution

The fractal picture that we have discussed in the previous sections has encountered an unreasonable resistance probably because it puts into question one of the fundamental hypotheses of cosmology like homogeneity. On the contrary, the concept of multifractal seems to be accepted in a smoother

way and this is quite curious (Jones et al., 1988; Martinez & Jones, 1990). Actually this probably happens for the wrong reasons. In fact we have the impression that some authors may consider multifractal as a sort of compromise in which scale invariant structures at all scales may coexist with homogeneity (Jones et al., 1988). This is actually not the case and multifractals are in contrast with homogeneity exactly like fractals. In fact the multifractal picture is a refinement and generalization of the fractal properties (Paladin & Vulpiani, 1987; Benzi et al., 1984). In the simple fractal case one refers to the properties of a set of points and one needs only an exponent. In the more complex case, when the scaling properties can be different for different regions of the system, one has to introduce a continuous set of fractal indexes to characterize the system (the multifractal spectrum). One refers to this case with the term "multifractality". The discussion that we have presented in the previous section, was meant to distinguish between homogeneity and scale invariant properties and, for this purpose, it is perfectly appropriate even if the galaxy distribution would be multifractal. In this case the correlation functions we have considered would correspond to a single exponent of the multifractal spectrum, but the issue of homogeneity versus scale invariance (fractal or multifractal) remains exactly the same.

In the previous section we have established that the basic characteristic of the observable galaxy distribution is that the two point *number – number* correlation function is a power law up to the sample limit for galaxy and cluster distributions:

$$G(r) = \langle n(r)n(0) \rangle \sim r^{-(3-D)} \quad (3.110)$$

A second important observational feature is the galaxy mass function: this function determines the probability of having a mass in the range between  $M$  and  $M + dM$  per unit volume, and can be described by the Press-Schechter function that shows a power law behaviour followed by an exponential cut-off for large masses (Press & Schechter, 1974):

$$n(M)dM \sim M^{\delta-2} \exp(-(M/M^*)^{2\delta}) dM \quad (3.111)$$

with  $\delta \sim 0.2$ . Hence the distribution of masses is also characterized by a power law corresponding to self-similarity of different nature. These two properties are naturally unified by the concept of multifractality. This concept naturally arises if the distribution of matter, as given by both positions and masses, has self similar properties. Indeed, masses of different galaxies can differ by as much as a factor  $10^6$  and it is important to include these mass values in order to describe the entire matter distribution and not just the galaxy positions.

The distribution of visible matter is described, in a certain sample of depth  $R_s$ , by the density function:

$$\rho(\vec{r}) = \sum_{i=1}^N m_i \delta(\vec{r} - \vec{r}_i) \quad (3.112)$$

where  $m_i$  is the mass of the  $i$ -th galaxy. This distribution corresponds to a measure defined on the set of points which have the correlation properties described by Eq.3.110. It is possible to define the normalized density function:

$$\mu(\vec{r}) = \sum_{i=1}^N \mu_i \delta(\vec{r} - \vec{r}_i) \quad (3.113)$$

with  $\mu_i = m_i/M_T$  and  $M_T = \sum_{i=1}^N m_i$ . The quantity  $\mu(\vec{r})$  is dimensionless. Suppose that the total volume of the sample consists of a 3-dimensional cube of size  $L$ . We divide this volume into boxes of linear size  $l$ . We label each box by the index  $i$  and construct for each box the function:

$$\mu_i(\epsilon) = \int_{i-th\ box} \mu(r) dr \quad (3.114)$$

Figure 8: Example of a multifractal measure: the (x,y) coordinates are the spatial ones, while the z axis is the measure. The support of the measure is itself a fractal.

where  $\epsilon = l/L$ . In the case of a MF distribution if in the  $i$ -th box there is a singularity of type  $\alpha$  then in the limit  $\epsilon \rightarrow 0$ , the measure goes as:

$$\mu_i(\epsilon) \sim \epsilon^{-\alpha(\vec{x})} \quad (3.115)$$

In Eq.3.115 the exponent  $\alpha(\vec{x})$  (a sort of local fractal dimension) fluctuates widely with the position  $\vec{x}$ . For an homogeneous mass distribution, with a uniform density,  $\alpha = 3$ , while for a simple fractal with dimension  $D$ ,  $\alpha = D$ . In general we will found several boxes with a measure that scales with the same exponent  $\alpha$ . These boxes form a fractal subset with dimension  $f(\alpha)$ . Hence the number of boxes that have a measure that scale with exponent in the range  $[\alpha, \alpha + d\alpha]$  vary with  $\epsilon$  as:

$$N(\alpha, \epsilon)d\alpha \sim \epsilon^{-f(\alpha)}d\alpha \quad (3.116)$$

The spatial density of these singularities in the total volume of the sample  $L^3$  vary with  $\epsilon$  as:

$$n(\alpha, \epsilon)d\alpha \sim \epsilon^{3-f(\alpha)}d\alpha \quad (3.117)$$

The function  $f(\alpha)$  is usually (Paladin & Vulpiani, 1987) a single humped function with the maximum at:

$$\max_{\alpha} f(\alpha) = D(0) \quad (3.118)$$

where  $D(0)$  is the dimension of the support. In the case of a single fractal, the function  $f(\alpha)$  is reduced to a single point:  $f(\alpha) = \alpha = D(0)$

A characteristic value of the spectrum is  $\alpha_{min}$  and the corresponding value of  $f(\alpha)$ . This formalism is suitable for the analysis of any distribution, even for a regular (analytic) one, because it is completely general and without any a priori assumption. The MF implies a strong correlation between spatial and mass distribution (Fig.8) so that the number of objects with mass  $M$  in the point  $\vec{r}$  per unit volume  $\nu(M, \vec{r})$ , is a function of space and mass and is not separable in a space density multiplied by a mass function (Binggeli et al., 1988). This means that we *cannot* express the number of galaxies  $\nu(M, x, y, z)$  lying in volume  $dV$  at  $(x, y, z)$  with mass between  $M$  and  $M + dM$  as:

$$\nu(M, x, y, z)dM dV = n(M)D(x, y, z)dM dV \quad (3.119)$$

Figure 9: The multifractal spectrum  $f(\alpha)$  derived from the *CfA1* redshift survey (Coleman & Pietronero, 1992). The strongest singularity in the distribution of visible matter is characterized by an exponent  $\alpha_{min} = 0.65$  and the corresponding fractal dimension is  $f(\alpha_{min}) \approx 0$ .

where  $D(x, y, z)$  is the density of galaxies of any luminosity. Moreover we cannot define a well defined average density, independent from sample depth as for the simple fractal case.

It can be shown (Sylos Labini & Pietronero, 1995b) that the mass function of a MF distribution, in a well defined volume, has indeed a Press-Schechter behaviour whose exponent  $\delta$  (Eq.3.111) can be related to the properties of  $f(\alpha)$ . Moreover the fractal dimension of the support is  $D(0) = f(\alpha_s) = 3 - \gamma$  (Eq.3.110). Hence with the knowledge of the whole  $f(\alpha)$  spectrum one obtains information on the correlations in space as well as on the mass function.

If one wants to perform the analysis of the mass distribution of galaxies, obviously one needs to know the density distribution  $\rho(\vec{r})$ . Usually an estimate of this quantity is obtained assigning to each galaxy a mass proportional to its luminosity. Clearly this is a crude approximation, however a better relation between luminosity and mass should not change the MF nature of the mass distribution, if it is present in the sample, but only the parameters of the spectrum. The analysis, carried out on *CfA1* redshift surveys (Coleman & Pietronero, 1992), provide unambiguous evidence for a MF behavior. In Fig.9 is shown the  $f(\alpha)$ -spectrum derived by Coleman & Pietronero (1992). This result has very deep physical implications and can naturally resolve several puzzling problems, as we shall see in the following.

#### 3.4.4.1. Luminosity function

The differential Luminosity Function (LF),  $\phi(L)dL$ , gives the probability of finding a galaxy with luminosity in the range  $[L, L + dL]$  in the unit volume ( $Mpc^{-3}$ ) and it can be described by the Schechter luminosity function (Binggeli et al., 1988):

$$\phi(L) = \phi^*(L/L^*)^{-\alpha} \exp(-L/L^*) \quad (3.120)$$

where  $L^*$  is the cut-off,  $\phi^*$  is the normalization constant and  $\alpha$  is the exponent. In the literature (see for a review Binggeli et al., 1988) one finds several methods to determine the LF for field galaxies and cluster galaxies. The standard analysis of the LF is based on the assumptions that the distribution of galaxies is homogeneous and that galaxian luminosities are not correlated with spatial locations. These a priori assumptions lead to several problems in the analysis of the LF. In

fact, due to the strong inhomogeneities present in all the available redshift samples, the average density, i.e. the amplitude of the LF, is not well defined and hence the classical methods intend to separate the determination of the shape and the amplitude of the LF. In particular, the so called *inhomogeneity-independent methods* have been developed with the intent to determine only the shape of the LF. These avoid the problems due to the presence of strong inhomogeneities and the sample depth dependence of the amplitude (for a more detailed discussion see Sylos Labini & Pietronero 1995b).

If the distribution is a simple fractal, then the amplitude  $\phi^*$  of the LF scales as  $\sim r^{-(3-D)}$  as the average density (Eq.3.92). The LF in this case can be written as:

$$\phi = Ar^{-(3-D)}(L/L_*)^{-\alpha} \exp(-L/L_*) dL/L_* \quad (3.121)$$

where  $A$  is a constant, and we assume the functional form in  $L$  as an experimental fact.

If we want to consider the whole luminosity-space distribution we have to analyze the MF case. In this case the amplitude scales again as  $r^{-(3-D)}$ , but we can obtain information also on the functional form of the LF. In fact if the whole mass distribution is MF, the exponent of the mass function is uniquely determined by the shape of the  $f(\alpha)$ -spectrum. Qualitatively we can say that also the exponent of the LF is linked to the multifractal spectrum, if one transforms masses into luminosities. However from the data analysis one determines the  $f(\alpha)$  spectrum of the luminosity distribution rather than of the mass one, so that one can determine in a different way from the classical methods, the exponent of the luminosity function (Sylos Labini & Pietronero, 1995b)

Moreover if the distribution is MF it can be shown (Coleman & Pietronero, 1992) that the brightest luminosity  $L_{max}$  in a sample is related to its depth  $R_s$  by the relation:

$$L_{max} \approx R_s^{3-\alpha_{min}} \quad (3.122)$$

Also this relation can be tested in real redshift surveys, even if it requires a high statistics because it deals with strongly fluctuating quantities.

#### 3.4.4.2. Number counts

The count-magnitude relation (i.e. the number of galaxies with apparent magnitude lower than  $m$  versus  $m$ ) is used to test the uniformity of galaxy distribution in space at small distances, and the galaxy luminosity properties in deeper samples. We show the behaviour of count-magnitude relation for an homogenous, fractal and multifractal distribution in the three dimensional Euclidean space. Indeed at small distances it is possible to neglect the effect of space-time evolution.

In the case of homogenous distribution the number of galaxies with magnitude less than  $m$ , from Eq.3.71, goes as:

$$\log(N(< m)) \sim \alpha m + const \quad (3.123)$$

with  $\alpha = 0.6$ . If the distribution is fractal with dimension  $D$ , it is simple to show that (Peebles 1993):

$$\alpha = \frac{D}{5} \quad (3.124)$$

This relation generalizes the previous one and, if  $D = 3$ , we readily obtain  $\alpha = 0.6$ . It seems from Eq.3.124 that knowing the luminosity properties of galaxy distribution as expressed by the count-magnitude relation, it is possible to reconstruct the fractal exponent of the space distribution. This is possible only if there is not any correlation between space locations and luminosities of galaxies. This is not the case for multifractal distributions. Indeed in this case it is possible to show (Sylos Labini & Pietronero, 1995b, c) that the exponent  $\alpha$  is not simply related to the fractal

dimension of the support as in Eq.3.124, but it is a complex function of the whole multifractal spectrum. In this case the exponent is greater than the value given by Eq.3.124.

In any case to check the validity of Eq.3.124 in real data, one has to consider *volume limited subsamples*, where it is possible to study the correlation properties without any bias, rather than magnitude limited subsamples as usually done (see *section 4.1.4.*).

#### 3.4.4.3. Magnitude-redshift relation

Taking the Schechter function (Eq.3.120) as the correct LF in the case of homogenous distribution, the joint distribution in galaxy redshifts  $z$  and energy flux densities  $f$  observed in region of sky with unit solid angle is (Peebles, 1993)

$$\frac{d^2N}{dzdf} = \int \phi_*(L/L_*)^{-\alpha} e^{-L/L_*} dL/L_* r^2 dr \delta(z - H_0 r) \delta(f - L/4\pi r^2) \quad (3.125)$$

This ignores the effect of space curvature. Integrating Eq.3.125 one obtains

$$\frac{d^2N}{dzdf} = \frac{4\pi}{L_*} \left( \frac{c}{H_0} \right)^5 z^4 \phi(kz^2) \equiv D(z, f) \quad (3.126)$$

where  $k = 4\pi f c^2 / H_0^2 L_*$ . The mean value of the redshifts of galaxies with given flux  $f$  is:

$$\langle z \rangle = \frac{\int z D(z, f) dz}{\int D(z, f) dz} = A_* f^{-1/2} \quad (3.127)$$

( $A_*$  is constant) or in magnitude ( $f \sim 10^{-0.4m}$ ):

$$\langle z \rangle = 10^{0.2m+a_*} \quad (3.128)$$

where  $a_*$  is a constant. In the case of a fractal distribution we can use the form of the LF given by Eq.3.121. Inserting this expression in Eq.3.125 one obtains instead of Eq.3.126:

$$D(z, f) \equiv \frac{d^2N}{dzdf} \sim z^{1+D} \quad (3.129)$$

and instead of Eq.3.128

$$\langle z \rangle = 10^{0.2m+a_f} \quad (3.130)$$

for every value of  $D$  ( $a_f$  is constant for fixed depth, but it depends on the sample depth and it is related to the fractal dimension - Sylos Labini & Pietronero, 1995b). Hence this is the theoretical expectation in the case of a fractal distribution in a three dimensional Euclidean space that has to be checked by the data (see *section 4.1.2.*). The behavior of the  $\langle z \rangle - m$  relation has the same exponent in an homogenous and in a fractal distribution whit the constant different in the two cases: it is related to the behavior of the average density.

#### 3.4.4.4. Luminosity segregation

Dressler (1984), and Einasto & Einasto (1987) found that the brightest galaxies lie preferentially in dense environments, in the core of groups and clusters of galaxies. The fact that the giant galaxies are "more clustered" than the dwarf ones has given rise to the proposition that larger objects may correlate up to larger length scales and that the amplitude of the correlation function is larger for giants than for dwarfs. This effect actually is a consequence of multifractality. For example in Fig.8 one can see that the largest peaks of the distribution are located in largest clusters. This arises naturally, due to the self-similarity of matter distribution. Moreover the multifractality

Figure 10: The probability of making an irreversible aggregation during a collision is greater in denser regions (a) than in sparser ones (b)

implies that the largest peak has a lower fractal dimension than the smaller ones: a trend in this direction has been found (Giovanelli et al., 1986) in the steeper angular correlation function for the early type galaxies (elliptical) than for late type (spirals). We recall that the exponent of the angular correlation function is not changed by projection (for a more detailed discussion see Sylos Labini & Pietronero, 1995c).

#### 3.4.4.5. A simple stochastic model for the formation of a MF distribution

From a theoretical point of view one would like to identify the dynamical processes that lead to such a MF distribution. In order to gain some insight into this complex problem we have developed a simple stochastic model (Sylos Labini & Pietronero, 1994a; 1994b; Sylos Labini et al.; 1995b) that includes the basic properties of the aggregation process and allows us to pose a variety of interesting questions concerning the possible dynamical origin of the MF distribution. The dynamics is characterised by some parameters, that have a direct physical meaning in terms of cosmological processes. In this way we can relate the input parameters of the dynamics to the properties of the final configuration and produce a sort of phase diagram.

In our model the formation of structures proceeds by merging of smaller objects. When two particles collide, in order to form a bound state, they have to dissipate a certain amount of energy. The basic physical mechanism responsible for energy dissipation in collisionless and pressureless dustlike particles, interacting only via gravitational force, is the *dynamical friction*: a test particle moving through a cloud of other background particles undergoes a systematic deceleration effect due to the gravitational scattering (Chandrasekhar, 1943). Due to this effect of energy dissipation, the aggregation process depends on the *environment* in which it takes place, and it is more efficient in denser region. Hence when two particles collide, they have a probability  $P_a$  of irreversible aggregation and probability  $1 - P_a$  to scatter (Fig.10). We find that the environment dependence of the dynamical friction, and then of the aggregation probability, *breaks the spatial symmetry* of the aggregation process and it is one of the fundamental elements that can give rise to a fractal (and multifractal) distribution.

This model shows an asymptotic fractal distribution in a certain range of the dynamical parameters. The non-linear dynamics leads spontaneously the self-similar (multifractal) fluctuations of the asymptotic state, so that there is not any crucial dependence on initial conditions. The fractal dimension of the asymptotic state depends only on the parameters of the non-linear dynamics. We find therefore that the necessary ingredients for a dynamics in order to generate a fractal (multifractal including masses) distribution are the breaking of the spatial symmetry, and the Self-Organised nature of the dynamical mechanism. This is why the formation of a multifrac-

tal distribution is not due to an amplification of the small amplitude initial fluctuations, but the generation of such complex structures is intrinsically generated by the non linear dynamics, that has an asymptotic critical state.

#### 3.4.4.6. Gravitational lensing in a fractal distribution

The usual approach to the study of the gravitational lensing statistics is based on the assumption that the matter distribution is homogenous (Turner et al., 1984). Baryshev et al., (1995b) considered the gravitational lensing effect inside a fractal distribution of matter showing that there are large differences between this case and the case of homogenous distribution of total (visible and dark) matter (Fig.11.).

In the case of point mass lenses the differential optical depth is (Baryshev et al., 1995b):

$$d\tau_g = 2\pi AR_H^{(D-1)} R_g \frac{(x_S x_L^{(D-2)} - x_L^{(D-1)})}{x_S} dx_L \quad (3.131)$$

where the average density scales as (see *section 3.4*):

$$\langle n \rangle = \frac{3A}{4\pi} R^{D-3} \quad (3.132)$$

and  $D$  is the fractal dimension,  $R_g = 2GM_*/c^2$  is the lens gravitational radius, and  $x_S$  and  $x_L$  are source and lens dimensionless distances ( $R_H$  is the Hubble radius). Integrating this equation along the line of sight to the source one obtains the total optical depth of lensing between the observer in the origin of the coordinate system and the source at distance  $R_s = x_S R_H$ :

$$\tau_g = 2\pi AR_H^{(D-1)} R_g \frac{x_S^{(D-1)}}{D(D-1)} \quad (3.133)$$

The differential optical depth, in the case of isothermal galaxies is :

$$d\tau_g = A\sigma_0 R_H^D \frac{(x_S - x_L)^2 x_L^D}{x_S^2} dx_L \quad (3.134)$$

where  $\sigma_0$  depends on the profile used (Turner et al., 1984) for modeling the isothermal galaxy. As in the previous case we calculate the total optical depth of lensing:

$$\tau_g = AR_H^D \sigma_0 x_S^D \frac{2}{D(D+1)(D+2)} \quad (3.135)$$

Hence, in principle, it is possible to distinguish between the homogeneous and the fractal case, even if the currently available data are still scarce. Hopefully, in the next years there will be new data on gravitational lenses so that, in principle, it will be possible to obtain information, using quasars, essentially over the entire causally connected universe.

### 3.4.5 Newtonian fractal models

The idea of modeling the Universe as a hierarchical structure is an old concept, that now can be discussed from the point of view of Fractals. The concept of fractal or self-similarity has behind it the same scaling idea of the old hierarchical clustering. The first person who introduced this idea was Fournier D'Albe (1907). Charlier (1908, 1922) applied this idea to a Universe model in order to explain Olber's paradox: in fact (see *section 3.1.2.*), if the fractal dimension is less than two, this paradox is naturally avoided without introducing the redshift. We have discussed in *section*



Figure 11: (a) the differential optical depth, normalized to the total one, for the homogenous (dotted line) and fractal ( $D = 2$ , solid line) case for point masses lens, versus the distance of the lens  $x_L$ : the source is at  $z_s = 0.3$ . (b): the total optical depth for the homogenous (dotted line) and fractal ( $D = 2$ , solid line) case for point masses lens, versus the normalized distance of the source. (c) The same of (a) but for the case of isothermal galaxies. (d) The same of (b) but for the case of isothermal galaxies. The difference between the fractal and the homogeneous case is appreciable (from Baryshev et al., 1995b).

3.1.2. that if the fractal dimension is one, the gravitational paradox, in the Newtonian theory of gravitation also is resolved, as the virial velocity inside the cluster does not depend upon  $r$ .

G. de Vaucouleurs (1970) proposed a hierarchical cosmology as a way of explaining his finding that galaxies seem to follow an average density power law with negative slope. Haggerty & Wertz (1971, 1972), developed further the Newtonian hierarchical models and proposed some possible observational tests. The hierarchical model proposed by Wertz (1970) was conceived at a time when fractal ideas had not yet appeared, but he assumed the De Vaucouleurs' density power law as the fundamental empirical fact to be taken into account in order to develop a hierarchical cosmology. He described a "*polka dot model*", that is a structure in discrete hierarchy, where the elements of a clusters are all of the same mass and are distributed regularly in such a way that we can call now deterministic fractal structure. He showed that regular hierarchic models are fully defined by a single parameter - the fractal dimension.

### 3.4.6 General Relativity Fractal Models

The standard approach to cosmology using General Relativity (GR) assumes that a well defined mean density exists in the observable Universe. We have discussed in *section 3.4.3.* that it is clear that the large-scale structure of the Universe does not show itself as a smooth and homogenous distribution of luminous matter, but it has a well defined fractal nature. In other words the mean density scales according to de Vaucouleurs' formula (1970):

$$\langle \rho \rangle \sim r^{-(3-D)} \quad (3.136)$$

The exponent of this law has a fundamental physical meaning and needs to be explained by the theory. The general solution of Einstein's equation for spherically symmetric dust in comoving coordinates is the Tolman solution (Tolman, 1934). This solution is spherical symmetric about *one* point, so that a cosmological model based on this solution does not consider the equivalence of all points in the Universe. For this reason Ribeiro (1992a, 1992b, 1993) constructs a model, called the *Swiss cheese model*, with an interior solution provided by the Tolman metric surrounded by a Friedmann space-time: with this scheme it is possible to save the equivalence of all points in the Universe. Such a model needs to solve the junction condition between the two metrics.

The Tolman metric with  $\Lambda = 0$  and  $G = c = 1$  may be written as:

$$ds^2 = dt^2 - \frac{R'^2}{f^2} dr^2 - R^2 d\Omega^2 \quad (3.137)$$

with  $r \geq 0$  and  $R(r, t) \geq 0$ , where  $d\Omega^2$  is the differential solid angle and  $f(r)$  is an arbitrary function. Einstein's field equations for this metric reduce to a single equation:

$$2R \frac{\partial R}{\partial t} + 2R(1 - f^2) = F \quad (3.138)$$

where  $F(r)$  is another arbitrary function. There are three classes of Tolman solutions ( $f^2 = 1$ , parabolic ;  $f^2 > 1$  hyperbolic and  $f^2 < 1$  elliptic) that correspond respectively to flat, open and closed Friedmann models. In order to obtain fractal solutions in the Tolman model, Ribeiro had performed some numerical calculations, and the criteria for choosing and accepting the solutions are: the linearity of the distance-redshift relation for  $z < 1$  with the Hubble constant in the range  $40 - 100 kmsec^{-1} Mpc^{-1}$ , the constraints of the fractal dimension to be in the range  $[1, 2]$  and the obedience to the de Vaucouleurs' density power law  $\langle \rho \rangle \sim d_l^{-\gamma}$ , where  $d_l$  is the luminosity distance. The reason for choosing this distance is that this is an observable quantity.

Ribeiro found fractal behaviour in hyperbolic models choosing appropriately the arbitrary functions of the Tolman model. In this model  $H_0 \sim 80 kmsec^{-1} Mpc^{-1}$  and  $D = 1.4$  up to

$d_l \sim 270Mpc$ . It seems that this solution is highly dependent on the parameters used, whose change can produce a qualitative change in its behaviour. If a Friedmann metric is joined to the Tolman-fractal solution it is found that  $\Omega_0 \sim 0.002$ . It is interesting that it is not a good modeling of a possible crossover to homogeneity in a fractal system, because Ribeiro showed that the Friedmann metric looks very inhomogeneous at larger scales when measured along the past light cone: this implies that this external solution is not really mandatory. Clearly this is a simple model but it represents a first attempt to model a fractal Universe within the GR framework. In particular it is found that the idea of a vanishing global density is compatible with the GR solution. In fact the average density is made not at a space-like hypersurface of constant time, but the average is calculated along the backward null cone, and in this case the observable density can go to zero. This model provides a description for the possible fractal structure, but it does not provide an answer to the question of the origin of such structure.

### 3.4.7 Field Fractal Models

As a possible way to construct a new type of cosmological model we consider here the so-called Field Fractal model based on a relativistic tensor field ( $\psi^{ik}$ ) gravitation theory in flat Minkowski space-time ( $\eta^{ik}$ ). The widespread opinion on the full coincidence of Tensor Field Theory (TFT) and General Relativity (GR) (e.g. Misner, Thorn & Wheeler, 1973) is based on the assumption of the uniqueness of the energy-momentum tensor (EMT) of the gravitational field, which was used for the iteration procedure. However, in the framework of Lagrangian formalism of relativistic field theory, the EMT of any field is not defined uniquely, but can be transformed (e.g. Landau & Lifshiz, 1973; Bogolubov & Shirkov, 1976):

$$T^{ik} \Rightarrow T^{ik} + \Phi^{ikl}_{,l} \quad (3.139)$$

for  $\Phi^{ikl} = -\Phi^{ilk}$ , and one needs additional physical restrictions to choose the final form of the EMT; for example such conditions as a positive energy, a EMT with null trace, or some symmetries. It is apparent that different EMTs would lead to different nonlinear theories of gravitation.

To illustrate the above discussion let us consider the nonlinear generalization of Poisson's equation for the case of distributed source, describing the negative and the positive energy density of the gravitational field:

$$\Delta\varphi = -\frac{(\nabla\varphi)^2}{c^2} \quad (3.140)$$

and

$$\Delta\varphi = +\frac{(\nabla\varphi)^2}{c^2} \quad (3.141)$$

The solution of Eq.3.140 is:

$$\varphi = c^2 \ln \left( 1 - \frac{GM}{rc^2} \right) \quad (3.142)$$

$$\frac{d\varphi}{dr} = \frac{GM}{r^2(1 - GM/rc^2)} \quad (3.143)$$

whereas the solution of Eq.3.141 is:

$$\varphi = -c^2 \ln \left( 1 + \frac{GM}{rc^2} \right) \quad (3.144)$$

$$\frac{d\varphi}{dr} = \frac{GM}{r^2(1 + GM/rc^2)} \quad (3.145)$$

From Eq.3.142 - 3.145 we see two possible ways to construct the nonlinear TFT. The first one, based on the negative energy density of the gravitational field, leads to the infinite gravity force at the finite distance  $R_g = GM/c^2$ . The second way, based on positive energy, gives TFT without singularity.

In GR the energy density of the gravitational field is a poorly defined concept as a consequence of the geometrical interpretation of gravity (pseudotensor character of the gravitational EMT). For example, Landau-Lifshiz (1973) pseudotensor gives  $T^{00} = -7(\nabla\varphi_N)^2/8\pi G$  but the Grischuk-Petrov Popova (1984) gives  $T^{00} = -11(\nabla\varphi_N)^2/8\pi G$  for the spherically symmetric static (hereafter SSS) weak field in harmonic coordinates.

In the TFT approach there is a real tensor quantity for the energy density of the gravitational field. For example the canonical EMT gives  $T^{00} = +(\nabla\varphi_N)^2/8\pi G$  for an SSS weak field and it corresponds to a quantum description of the gravitational field as an aggregate of gravitons in flat space-time. Gravitons are massless particles, i.e. some kind of matter in space-time, which carry positive energy and momentum in the space-time. Besides the sum of two tensor  $\psi^{ik} + \eta^{ik} = g^{ik}$  is not a metric tensor because the covariant components of this tensor  $\psi_{ik} + \eta_{ik} = g_{ik}$  provide the mixed components  $\psi_k^i + \eta_k^i = g_k^i \neq \delta_k^i$  and the trace  $g^{ik}g_{ik} = 4 + 2\psi + O(\psi_{ik}^2) \neq 4$ . It means that the TFT is a scalar-tensor theory, and not a pure tensor one, and hence it includes spin 2 and spin 0 gravitons.

Hence the first hypothesis in this approach is to choose the TFT as the gravitation theory. Let us consider the relativistic symmetric field  $\psi^{ik}$  in Minkowski's space-time  $\eta^{ik}$ . As the really observed gravitational fields are the weak ones ( $|\varphi| \ll c^2$ ), it is natural to begin the construction of the TFT for the weak field case. In this case we have a very close analogy with the electromagnetic field and can use the standard Lagrangian formalism of the relativistic field theory (below we utilize the notation of the Landau-Lifshiz textbook). We begin with the action integral in the form:

$$S = S_{(g)} + S_{(int)} + S_{(p)} = \frac{1}{c} \int (\Lambda_{(g)} + \Lambda_{(int)} + \Lambda_{(p)}) d\Omega \quad (3.146)$$

where  $(g)$ ,  $(int)$  and  $(p)$  indicate respectively the gravitational field, the interaction and the particles parts of the actions and Lagrangians. The Lagrangians are given by the following expressions:

$$\Lambda_{(g)} = -\frac{1}{16\pi G} (2\psi_{nm}^n \psi_{,l}^{lm} - \psi_{lm,n} \psi^{lm,n} + 2\psi_{ln}^l \psi^{,n} + \psi_{,l} \psi^{,l}) \quad (3.147)$$

$$\Lambda_{(int)} = -\frac{1}{c^2} \psi_{lm} T_{(p)}^{lm} \quad (3.148)$$

$$\Lambda_{(p)} = -\eta_{ik} T_{(p)}^{ik} \quad (3.149)$$

It has been shown by Kalman (1961) and Thirring (1961) that the total EMT of the system consists of three parts, which correspond to those of the action integral in Eq.3.146:

$$T_{(\Sigma)}^{ik} = T_{(g)}^{ik} + T_{(int)}^{ik} + T_{(p)}^{ik} \quad (3.150)$$

where the canonical EMT of the gravitational field for Lagrangian Eq.3.147 has the form:

$$T_{(g)}^{ik} = \frac{1}{8\pi G} \left( (\psi^{lm,i} \psi_{lm}^k - \frac{1}{2} \eta^{ik} \psi_{lm,n} \psi^{lm,n}) - \frac{1}{2} (\psi^{,i} \psi^{,k} - \frac{1}{2} \eta^{ik} \psi_{,l} \psi^{,l}) \right) \quad (3.151)$$

the interaction EMT is

$$T_{(int)}^{ik} = \frac{2}{c^2} T_{(p)l}^i \psi^{lk} - \frac{1}{c^2} T_{(p)}^{ik} \psi_{lm} u^l u^m \quad (3.152)$$

the point particles EMT is

$$T_{(p)}^{ik} = \sum_a m_a c^2 \delta(\vec{r} - \vec{r}_a) \left(1 - \frac{v_a^2}{c^2}\right)^{1/2} u_a^i u_a^k \quad (3.153)$$

Therefore the nonlinear EMT must include the interaction Lagrangian in the form:

$$\Lambda_{(int)} = -\frac{1}{c^2} \psi_{lm} T_{(\Sigma)}^{lm} \quad (3.154)$$

The weak field condition allows us to use the linear approximation as the first step and then to make nonlinear correction (Post-Newtonian - hereafter PN- TFT). The variation of the gravitational potentials in the action integral Eq.3.146, where for fixed sources in the PN approximation we can use the interaction Lagrangian in Eq.3.154, yield the PN field equation in the form:

$$-\psi_l^{ik,l} + \psi_l^{il,k} + \psi_l^{kl,i} - \psi^{,ik} - \eta^{ik} \psi_{,lm}^{lm} + \eta^{ik} \psi_{,l}^l = \frac{8\pi G}{c^2} T_{(\Sigma)}^{ik} \quad (3.155)$$

Eq.3.155 automatically requires the conservation of the total EMT and leads to the equation of motion for particles in the form  $T_{(\Sigma),k}^{ik} = 0$ . The field equations are invariant (for fixed sources) under the gauge transformation  $\psi^{ik} \Rightarrow \psi^{ik} + \theta^{i,k} + \theta^{k,i}$  and one achieves the Hilbert gauge in the form  $\psi_{,k}^{ik} = \frac{1}{2} \psi^{,i}$ . In this case the field equation (Eq.3.155) becomes:

$$\square \psi^{ik} = \frac{8\pi G}{c^2} \left( T_{(\Sigma)}^{ik} - \frac{1}{2} \eta^{ik} T_{(\Sigma)} \right) \quad (3.156)$$

In the case of an SSS weak field the first approximation EMT has the very simple form  $T_{(\Sigma)}^{ik} = \text{diag}(\rho_0 c^2, 0, 0, 0)$  and the solution of Eq.3.156 is the Birkhoff's potential:

$$\psi^{ik} = \varphi_N \text{diag}(1, 1, 1, 1) \quad (3.157)$$

where  $\varphi_N$  is the Newtonian potential, i.e.  $\varphi_N = -GM/r$  outside the gravitating body.

Using the SSS solution Eq.3.157 and the corresponding EMT expression Eq.3.151, 3.152, 3.153 we find the total energy densities of the system from Eq.3.150:

$$T_{(\Sigma)}^{00} = T_{(p)}^{00} + T_{(int)}^{00} + T_{(g)}^{00} = (\rho_0 c^2 + e) + \rho_0 \varphi_N + \frac{1}{8\pi G} (\nabla \varphi_N)^2 \quad (3.158)$$

where  $(\rho_0 c^2 + e)$  is the rest mass and the kinetic energy density,  $\rho_0$  is the interaction energy density,  $\nabla \varphi_N^2 / 8\pi G$  is the energy density of the gravitational field. The total energy of the system will be:

$$E_{(\Sigma)} = \int T_{(\Sigma)}^{00} dV = E_{(0)} + E_{(k)} + E_{(p)} \quad (3.159)$$

where  $E_{(0)} = \int \rho_0 c^2 dV$  is the rest-mass energy,  $E_k = \int (e) dV$  is the kinetic energy, and  $E_p$  is the classical potential energy that equals the sum of the interaction and the gravitational field energy:

$$E_p = E_{(int)} + E_{(g)} = \int \left( \rho_0 \varphi_N + \frac{1}{8\pi G} (\nabla \varphi_N)^2 \right) dV = \frac{1}{2} \int \rho_0 \varphi_N dV \quad (3.160)$$

Up to now TFT has been developed only for weak field approximation because there is no exact expression for the total EMT (Eq. 3.150) in the case of the strong field. But in TFT there is the possibility to consider the case of weak force ( $\nabla \varphi \rightarrow 0$ ) while  $\varphi \rightarrow c^2/2$ . It is just what we have for the cosmological problem. In this case we can get some qualitative results from post Newtonian TFT. Let us consider the case of a static homogeneous ( $\rho = \text{const}$ ) dust-like cold

matter ( $p = 0, e = 0$ ) distribution within infinite space. Inserting Eq.3.158 in the field equation Eq.3.156 and taking into account the traceless of the field and interaction EMTs, we get the equation for the  $\phi = \psi^{00}$  component in the form

$$\Delta\varphi = 4\pi G \left( \rho + 2\rho \frac{\varphi}{c^2} + \frac{2(\nabla\varphi)^2}{8\pi G c^2} \right) \quad (3.161)$$

In our case the main terms in the right-hand side of Eq.3.161 are the positive rest mass density  $\rho$  and the negative interaction mass density  $(2\rho\varphi)/c^2$ , because for  $\varphi \rightarrow \text{const}$  its gradient  $\nabla\varphi \rightarrow 0$ . Hence we can consider the following simple equation

$$\Delta\varphi - \frac{8\pi G\rho\varphi}{c^2} = 4\pi G\rho \quad (3.162)$$

Eq.3.162 is equivalent to Einstein's equation Eq.3.7:

$$\Delta\varphi - \lambda\varphi = 4\pi G\rho_0 \quad (3.163)$$

Comparing Eq.3.162 and Eq.3.163 we conclude that  $\Lambda$  - *term* in TFT is

$$\lambda = \frac{8\pi G\rho}{c^2} \quad (3.164)$$

and it corresponds to the interaction EMT. According to Einstein (1917) the cosmological solution of Eq.3.163 with Eq. 3.164 will be

$$\varphi = -\frac{4\pi G\rho}{\lambda} = -\frac{c^2}{2} \quad (3.165)$$

The cosmological solution Eq.3.165 can be derived also as a limiting case ( $R \rightarrow \infty$ ) of the exact solution of Eq.3.162 for a finite ball radius  $R$  (see Baryshev & Kovalevskii, 1990). So within TFT there is a natural static cosmological solution. Note that this solution is true for any mass distribution including a fractal one. For sufficiently small distances, we have a quasi Newtonian behavior of the gravitational potential, due to mass density fluctuations.

In the case of static Universe the cosmological redshift could be due to the global gravitational effect (see *section 2.2.3*). For having a linear redshift-distance relation the fractal matter distribution needs to have fractal dimension  $D = 2$  (Baryshev, 1981, 1994). As a first qualitative step to understand a new possibility in the cosmological solution, let us consider a generalization of the Baryshev & Kovalevskii (1990) expression for the total gravitational mass inside the homogeneous ball of radius  $R$  to the following form in the case of a fractal distribution with  $D = 2$ :

$$M(R) = M_H x \left( 1 - \frac{1 - e^{-4x}}{4x} \right) \quad (3.166)$$

where  $x = R/R_H$  is the metric distance  $R$  in the units of the Hubble radius  $R_H = (c^2\pi G\rho_0 R_0)/2$ ,  $\rho_0$  and  $R_0$  are the lower cutoff of the fractal structure (mass density and radius of average galaxy), and  $M_H = (c^2 R_H)/2G$  is the Hubble mass. From Eq.3.166 it follows for small distances ( $x \ll 1$ )  $M(R) = 2\pi\rho_0 R_0 R^2$  and for large ones ( $x \gg 1$ )  $M(R) = (c^2 R)/(2G)$ . By combination of equation for mass (Eq.3.166) and the equation for gravitational redshift (Eq.2.8) one can find the general redshift-metric distance relation

$$\mathcal{L}(z) = \frac{l}{R_H} = \log(1 + z) \quad (3.167)$$

where  $l = R$ . So for small  $z$  from Eq.3.167 we get the linear Hubble law

$$z = \frac{H_g l}{c} \quad (3.168)$$

where  $H_g$  may be called the gravitational Hubble constant

$$H_g = \frac{2\pi G \rho_0 R_0}{c} = 69 \text{ km s}^{-1} \text{ Mpc}^{-1} \quad (3.169)$$

and the value corresponds to mass density  $\rho_0 = 5.2 \cdot 10^{-24} \text{ g/cm}^3$  and radius  $R_0 = 10 \text{ kpc}$ . In the case of a gravitational cosmological redshift the angular and the bolometric distances are given by Eq.3.34. Hence the angular size-redshift relation has the form

$$\Theta(z) = \left( \frac{d}{R_H} \right) \frac{(1+z)}{\log(1+z)} \quad (3.170)$$

and the magnitude-redshift relation is

$$m_{bol}(z) = 5 \log \left( \frac{\log(1+z)}{(1+z)} \right) + \text{const} \quad (3.171)$$

Hence we have shown that within Field Fractal Models one can construct definite theoretical predictions but we emphasize that many questions are still open and need further development.

### 3.5 Other possibilities

Besides Steady State and Fractal models there are several other cosmological theories proposed as alternatives to the standard hot big bang model (see e.g. the review of Narlikar, 1987). The motivation for exploring alternatives comes from the existence of paradoxes in the standard model and from the necessity of analysis of possible initial hypotheses (see Fig.1). The Tired Light (TL) cosmological model was first suggested by Zwicky (1929) and it is based on the idea that the cosmological redshift is caused by some unknown physical process in which traveling photons continuously undergo an energy depletion or aging effect. So the cosmological redshift is given by Eq.2.15. Hubble and Tolman (1935) proposed several observational tests to permit a decision between recessional and tired-light causes for the cosmological redshift. These are the angular size-magnitude, the surface brightness-redshift and the number counts-magnitude relations. Later, many attempts have been made to construct a physical mechanism for the photon energy loss and to compare available observational data with the predictions of the TL model (see e.g. Jaakkala et al., 1979; La Violette, 1986; Vigier, 1988). They claimed that the TL model gives a good fit to observations. However, Sandage & Perlmutter (1991) have shown that the surface brightness of the giant elliptical galaxies have  $(1+z)^{-4}$  behavior while for the TL model the  $(1+z)^{-1}$  dependence was predicted. Nevertheless within the scope of the TL model one can introduce an evolution of the surface brightness to explain this observation. So the question about validity of the TL cosmology is still open and needs further crucial tests.

The idea about perfect symmetry between matter and antimatter in the Universe was proposed by Alfvén and Klein (Alfvén, 1989). The baryon symmetrical plasma was supposed to have been separated by hydromagnetic processes. For the modern version of the model see Stecker (1982). Future tests of the theory may come from observations of the cosmic neutrino background by underwater detectors. Note also such ideas as variable physical constant cosmologies (Dirac, 1937; Brans & Dicke, 1961; Troitskii, 1987), variable mass cosmology (Narlikar & Arp, 1993), chronometric cosmology (Segal, 1976). All these alternative cosmologies stimulate observational checking of possible theories and need construction of the crucial tests.

Figure 12: Hubble's law for nearby galaxies (from Peebles 1988b). The distance are determined using the infrared Tully-Fisher method:  $d = cz/H_0$ .

## 4 CONFRONTATION OF WORLD MODELS WITH OBSERVATIONS

In this section we consider the data that are currently available on classical and post-classical cosmological tests. For a more detailed discussion we refer the reader to more specific reviews such as Sandage (1987, 1988), Yoshii & Takahara (1988), Arp et al. (1990), Peebles et al. (1991), Coleman & Pietronero (1992), Coles & Ellis (1994), Efstathiou (1994). According to our classification (see *section 2*) we shall divide cosmological tests into two groups: parametric and crucial.

### 4.1 Parametric cosmological tests

Parametric tests permit the determinations of the parameters of each model. All these tests together can put strong constraints on the various parameters of a model so that as a whole these can be considered as a crucial test. We consider in detail classical parametric tests discussing the most recent observational data. Moreover we consider the post-classical parametric tests that, in the last few years, have been strongly developed from observations.

#### 4.1.1 Redshift-distance relation and the Hubble constant

The first main observational result is the linearity of the Hubble law at small distances (Fig.12), deep inside the inhomogeneity cell, in the range between  $2h_{50}^{-1}Mpc$  up to  $80h_{50}^{-1}Mpc$  (Sandage, 1986, 1991; Peebles, 1993). The uncertainty in the value of Hubble's constant does not affect the test of the linearity of the redshift-distance relation. In Fig.13 Hubble's law is shown for clusters (Mould et al., 1991) and in Fig.12 for individual galaxies (Peebles, 1988b). There are some evidences of large fluctuation in the linear  $z - d$  law in the Virgo and Great Attractor direction (Lynden Bell et al., 1998; Teerikorpi et al., 1992) so that further and more detailed studies are needed: the filled circles in Fig.13 represent clusters in the neighborhood of the Great Attractor.

The second important observational result is the clear tendency to the higher value of the Hubble constant. Recent observations converge to the value of  $H_0 \approx 80kmsec^{-1}Mpc^{-1}$ . Pierce



Figure 13: Hubble's law for clusters. The distance is determined using the Tully-Fisher relation (from Mould et al. 1991).

et al. (1994) found a value of  $87 \pm 7 \text{ km sec}^{-1} \text{ Mpc}^{-1}$  with the observations of three Cepheid variables in a galaxy of the Virgo Cluster; Schmidt et al. (1994) found  $\sim 75 \text{ km sec}^{-1} \text{ Mpc}^{-1}$  with the type II supernovae. Moreover Freedman et al. (1994) found  $H_0 = 80 \pm 17 \text{ km sec}^{-1} \text{ Mpc}^{-1}$  with the Cepheids method. However, according to Sandage (1994) the value of  $H_0$  is about  $40 - 50 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ .

Hence all cosmological models need to incorporate the linear  $z - d$  relation for small distance with, probably, a high value of  $H_0$ . The observed linear  $z - d$  relation is well fitted by the standard Friedmann, Steady State, Tired light models. But problems arise for SM, SSM and TL models when one attempts to take into account the strong fractal-like inhomogeneities at small distances where the  $z - d$  relation is linear.

Also there is a conflict between the large ages of globular clusters and distant radio galaxies in comparison with the age of the Universe that comes out in the Friedmann models (Dunlop et al., 1988; Chambers et al., 1990).

#### 4.1.2 Angular size versus redshift relation

After Hoyle's (1959) suggestion of testing cosmological models by the angular size-redshift relation there were a lot of papers devoted to this topic. The main problem of such type of test is the identification of the "*standard stick*" for which one knows reliably its non-variation with redshift, or its variation with a well known evolutionary scheme.

Up to now in optical waveband there are data only for relative small redshifts ( $z \leq 1$ ). In Fig.14 (from Djorgovski & Spinrad, 1981) one sees how the observed optical sizes of giant elliptical galaxies are fitted by Standard Friedmann, Steady State and non standard model with  $\Lambda$ -term models. In radio wavelengths the situation is in more doubt, because there is not a good standard stick such as the optical galaxy size (which is stable for a sufficiently long time). If one considers classical double radio sources are obtained the results shown in Fig.15 (from Kapahi, 1987). The recent discussion on double sources in the angular size test has been given by Nilsson et al. (1993) However for compact VLBI structures Kellerman (1993) found a deviation from the Euclidean behaviour of the angular size-redshift relation (Fig.16). The compact VLBI radio source is produced by relativistic jets, ejected from the central energy machine of the quasars, and

Figure 14: Optical angular sizes of giant galaxies versus redshift (from Djorgovski & Spinrad 1981).

Figure 15: Double radio sources angular size versus redshift (from Kapahi, 1987).

Figure 16:  $\Theta - z$  relation for VLBI structures (from Kellerman, 1993).

it is unlikely that such jets have a well defined linear size for all sources. This is why we need more observational information about the  $\Theta - z$  relation. The most promising could be joint radio and optical observations of a well defined sample of radiogalaxies.

### 4.1.3 Apparent magnitude-redshift relation

The slope of the magnitude-redshift ( $\langle m \rangle - z$ ) relation was found to be 5 within limits of the error (Sandage, 1988 for a review) (see Fig.17a,b). However this behaviour does not permit to discriminate between alternative models and in particular between fractal and homogeneous distributions, as they have the same slope (Eq.3.128 and Eq.3.130). A possible test in this direction can be the  $\langle m \rangle - z$  diagram at largest redshifts. The SM in fact predicts a correction to the  $5 \log(cz)$  law of first order in  $z$  (Eq.3.60). Such a measurement in principle permits the determination of  $q_0$ . In this case the problem is in estimating the evolutionary corrections for stellar evolution theory but no definitive answers have been obtained in the visible band (Sandage, 1988). In the near-IR there are several observations at large redshift: in this band the evolutionary corrections are predicted to be smaller than in the visible band. A summary of the K-data is given by Yoshii & Takahara (1988). Even in this case, as in the case of number counts, the situation is problematic due to the unknown evolutionary model that has to be taken into account. Clearly the argument is circular if we use the standard model predictions to prove that evolution has occurred without having a priori the proof that the standard model is correct.

### 4.1.4 Galaxy counts

The usual analysis of the galaxy counts is performed in magnitude limited subsamples. In Fig.18 the whole behaviour of  $\log(N(< m)) \approx \alpha \cdot m$  in the B-band is shown. It has been found (Shanks et al., 1984; Tyson, 1988; Broadhurst et al., 1990a,1990b; Colles et al., 1990; Lilly et al., 1991; Cowie et al., 1993) that  $\alpha \approx 0.6$  in the range  $15 < m < 18$  and  $\alpha \approx 0.45$  for  $m \sim 19$  up to  $m \sim 27$ . The fact that  $\alpha \approx 0.6$  at intermediate magnitudes has been invoked as a proof of homogeneity of the matter distribution (Peebles, 1993), while the subsequent change of slope has been interpreted as an effect of galaxy evolution, or a combined effect of galaxy and space-time evolution (Broadhurst et al., 1990a, 1990b; Yoshii, 1993; Lilly et al., 1991; Metcalfe et al., 1991;

Figure 17:  $\langle m \rangle - z$  relation in (a) the B band, and in (b) the K band. Thick and thin lines represent the models for evolving and nonevolving galaxies. Number beside each thin line is the value of  $q_0$  and numbers in parentheses are the values of  $q_0$  and  $z_F$  (from Yoshii & Takahara, 1988).

Figure 18: Differential galaxy number counts in the B-band. The symbols represent the count data taken from various surveys. For  $12 \lesssim m \lesssim 18$  the exponent is  $\alpha \approx 0.6$ , while in the range  $m > 19$   $\alpha \approx 0.45$ . This exponent has been established in magnitude limited samples (from Yoshii, 1993).

Figure 19: Galaxy number counts in the K-band. The counts are fitted with two separate power laws, with an exponent of 0.67 in the range  $10 < K < 16$  and 0.26 in the range  $18 < K < 23$  (from Gardner et al. 1993).

Lilly, 1993; Babul & Rees, 1992; Efstathiou et al., 1991).

In the infrared K-Band  $\alpha \sim 0.67$  for  $12 < K < 16$  and the slope changes at  $K \sim 18$  to  $\alpha \approx 0.26$  as shown in Fig.19. Hence this is in contrast with the B-band counts (Gardner et al. 1993, Mobasher et al., 1986; Jenkins & Reid, 1991). There are several biases and selection effects that should be taken into account in the study of the galaxy luminosity properties especially for the observations of faint galaxies (see *section 2.4*) and we refer to Yoshii (1993) and McGaugh (1994) for a detailed discussion.

If one considers the galaxy counts at small and intermediate redshifts ( $z < 0.2$ ) the galaxy and space-time evolution shall be neglected. In principle it should be possible to verify experimentally the relation  $\alpha = D/5$ , that has never been tested. To do this one needs to analyze the luminosity properties together with the correlation properties in volume limited subsamples, so that one does not introduce any bias due to observational selection effects (for a detailed discussion see Sylos Labini & Pietronero 1995c). For example, analyzing the *CfA1* redshift survey we have found that  $\alpha \sim 0.4$  for the whole magnitude limited catalog, while it is  $\alpha \sim 0.25 \approx D/5$  ( $D \approx 1.4 - 1.5$ ) for a volume limited subsample. Clearly in this case we have only a small range of magnitudes available.

In order to identify the effect of galactic and space-time evolution one has to clarify the space and luminosity properties and the possible correlation between them. In other words to isolate the evolutionary effect on the galaxy counts, certain properties of the spatial distribution of nearby galaxies need to be well known. If one defines evolution as the deviation from the Euclidean behavior ( $\alpha = 0.6$ ) there is the a priori assumption that the galaxies distribution becomes homogenous and that the space locations are not correlated with the luminosities of galaxies, while this seems not to be the case in the available samples.

#### 4.1.5 Other parametric tests

##### 4.1.5.1. Ages of objects

There are two methods to measure the age of the Universe  $T_0$ : (i) determination of the age of the oldest stars in a galaxy and (ii) determination of the age of the chemical elements. Regarding

the first, stellar evolution provides a stringent limit to the age of the oldest stars. These stars are observed in Globular Clusters and the age of these objects can be estimated by fitting isochrones to the Hertzsprung-Russell diagram. This method is fairly accurate but it is based on the theory of stellar evolution. It yields a typical age in the range  $(14 - 16)Gyr$  but there are some controversial estimates of  $20Gyr$ . Significantly lower ages are in conflict with the generally accepted theory of stellar evolution (Van der Berg, 1983, 1990; Sandage, 1988).

Radioactive decay ages are based on the radioactive isotopes of  $^{235}U$  in lunar and meteoric rocks. Typical estimates give  $T_0 \sim (9 - 16)Gyr$  but there is a big uncertainty depending on the adopted models of galaxy evolution and various other assumptions (Sybalisky & Schramm, 1981).

#### 4.1.5.2. Galaxy bulk flows

To study the distribution of mass and not only of the visible matter, one has to look not just to the spatial position of galaxies, but also at their peculiar velocities. In fact if peculiar velocities are generated by inhomogeneities in the distribution of matter, they can be used to estimate the total amount of mass. The total redshift of a galaxy will be in general

$$(1 + z) \approx (1 + z_H)(1 + z_p) \approx z_H + z_p + 1 \quad (4.1)$$

where  $z_H$  is the Hubble-redshift and  $z_p$  is the redshift due to the peculiar velocity along the line of sight and the last expression is true for small  $z$ . The problem is that one must measure the galaxy distance using some indicator other than the redshift. Knowing the redshift and the distance one readily obtains  $z_H$  and then  $v_p = c \cdot (z - z_H)$  from Eq.4.1. Actually there are two main methods of measuring distances that have been applied in the study of the peculiar velocities. The first is based on the Tully-Fisher relation (Tully & Fisher, 1977) for measuring the distances of spiral galaxies, while the second utilizes the  $D_n - \sigma$  relation (Lynden-Bell et al., 1988) for elliptical galaxies. In the last few years the peculiar velocities here have been measured for  $\sim 4000$  galaxies (Efsthathiou, 1994 for a review). Of course it is a very difficult task to produce an "homogenized" catalogue of the peculiar velocity field because the sampling of the peculiar velocity field is very patchy.

From the first controversial finding of Rubin et al. (1976) of a large-scale flow, called "Rubin-Ford"-effect, of  $600kmsec^{-1}$  in a direction nearly orthogonal to the CMBR dipole, a lot of evidences of very large scale coherent motions have been collected. Davis & Peebles (1983) suggested the existence of a systematic inflow of galaxies towards the center of the Virgo cluster, but Dressler et al. (1987) showed that the phenomena are more complicated than suggested by simple Virgo infall model. In fact there are important evidences of a systematic motion of  $\sim 600kmsec^{-1}$  with respect to the CMBR that suggests the existence for a large mass concentration, called the "Great Attractor" (GA), in the direction of the Hydra-Centaurus supercluster (Lynden-Bell et al., 1988).

An important kind of large-scale measurement of motions is the so-called "bulk" flow that is the estimate of the net velocity  $\langle V \rangle$  of galaxies within a large volume. There are evidences (Courteau et al., 1993) of large-scale coherent flows, and find a coherent motion on very large volume ( $\sim 150h^{-1}Mpc$ ) that implies an increase of  $\langle V \rangle$  on scales much larger than that of the GA. If the CMBR dipole is due to Doppler effect then the Local Group (LG) has a velocity of  $620kmsec^{-1}$  with respect to the rest frame represented by the CMBR. The origin of this motion can be due to some anisotropic mass fluctuation whose amplitude is expected (in an homogenous picture) to decrease with increasing scale. Hence the bulk flows of galaxies contained in very large volume should be at rest with respect to the CMBR. The results of Lauer & Postman show on the contrary that the LG motion relative to an Abell clusters sample is inconsistent with the velocity of the LG inferred from the CMBR dipole, and imply that the CMBR dipole anisotropy is generated by a very large scale mass concentration beyond  $100h^{-1}Mpc$ . Moreover Mathewson

et al.(1992a) found that there is not backside infall into the GA and that there is evidence of a bulk flow of  $600 km sec^{-1}$  in the direction of the GA on a scale at least  $60 h^{-1} Mpc$ . Willick (1990) found a bulk flow of  $450 km sec^{-1}$  in the opposite part of the sky and both these results suggested a bulk flow in the supergalactic plane over very large scale greater than  $130 h^{-1} Mpc$  (Mathewson et al., 1992b). Very recently Mathewson & Ford (1994) found that the flow is not uniform over the GA region and that it seems to be associated with a denser region that participates in the flow too.

From the data now available it emerges that the *full extent of the galaxies flows is still uncertain* and not detected and the origin of these large amplitude and coherent length peculiar motions is very unclear in the standard scenario. In fact it is very hard to reconcile these results with an homogenous picture, in which the bulk flows have to be small on large scales as the mass fluctuations have to be of small amplitude on the large scale. On the contrary in a fractal distribution, that is intrinsically inhomogeneous, there are fluctuations in the distribution of mass at all scales; in this case large-scale coherent flows are limited only by the the property of local isotropy that characterized a fractal structure and implies that the dipole (the net gravitational force) saturates beyond a certain scale (Sylos Labini, 1994).

#### 4.1.5.3. Gravitational lensing

The gravitational lensing effect is a new tool to test the very large-scale distribution of both visible and dark matter. One can use the lensing effect to map the total amount of mass of the lens-object, and use statistical arguments to constrain the large-scale distribution of matter. The cosmological applications of gravitational lensing and modern observational data have been considered by Blandford & Narayan (1992) and Refsdal & Surdej (1994). It has been shown by Baryshev et al. (1995b) that, in principle, it is possible to distinguish between the homogeneous and the fractal distribution of lenses in the Universe (see *section 3.4.4.6*). Unfortunately the currently available data are still scarce, but hopefully in the next few years there will be new data on the gravitational lenses so that, in principle, it will be possible to obtain information about large-scale structure of the matter distribution in the Universe essentially over the entire causally connected universe.

#### 4.1.5.4. Light element abundances

The determination of pregalactic (assumed primordial) abundances of light elements provides a probe to test the assumptions of cosmological models. At this aim, one needs to infer the cosmic abundances from the observed ones. Measurements of these abundances are carried out in our galaxy ( $^3He$  and  $^7Li$ ), and also in other galaxies ( $^4He$  and  $H$ ). For the  $D$ , the usual local ( $< 1 kpc$ ) measurements have been very recently, complemented by the first probable detection of  $D$  at a large redshift, i.e.  $z \sim 3.3$ , (Songaila et al., 1994). In any case, unless for this last measurement, the abundances of the elements that we observe today, are affected by chemical evolution of the galaxies. This is a complex process to model, as it involves the cycling of interstellar gas through generations of stars. Consequently, the inferred primordial abundances of the light elements are strongly model dependent results. The method for the detection of  $D$  is the same for either the measurements in the local interstellar medium or for the recent one at high redshift. In both cases, we study the absorption spectrum of an astrophysical light source (stars or quasars). The presence of  $D$  is detected through measurements of the isotopic shift of atomic HI lines (Boesgaard & Steigman, 1985). If we study the absorption spectra of high-redshift quasars, we can analyse the  $D$  abundance in a variety of very distant and fairly chemically unevolved environments. In this case one can bypass the need for the correction of the chemical evolution of the Galaxy and have a direct measurement of the ratio  $D/H$ . This last measurement has given a value of  $D/H$

as high as  $2.5 \cdot 10^{-4}$ , roughly  $\sim 10$  times the value observed in the interstellar medium. (Songaila et al., 1994a).

The abundance of  $^4\text{He}$  has been determined in a number of astrophysical sources: the atmospheres of young stars, the atmospheres of old stars, planetary nebulae, HII regions in our galaxy and in other galaxies, and in isolated extragalactic HII regions. Usually to deduce the primordial abundance from the observed one, models of stellar and galactic chemical evolution are needed. Low-metallicity, extragalactic HII regions reasonably to provide a "nearly" primordial sample of the helium abundance. This is determined from the analysis of the HII region emission spectra, as the intercept (at zero metallicity) of the regression line of the relation between helium and oxygen abundances. The primordial helium mass fraction  $Y_p$  is determined to be, with a 95% confidence limit,  $0.228 \pm 0.005$  (Pagel et al., 1992).

Lithium has been observed in hundreds of PopI stars of various ages, in a less number of PopII stars, in chondritic meteorites and in the interstellar gas. All of the abundances are based on measurements of the equivalent width of the Li I resonance doublet at  $6707.761 \text{ \AA}$  and  $6707.912 \text{ \AA}$ . As a star evolves, its surface Lithium is subject to destruction and dilution. However, the observed abundance in sufficiently metal-poor halo stars, seems, presumably, provide a nearly primordial sample. The inferred primordial abundance is bounded by  $10^{-10} < (^7\text{Li}/H)_p < 8 \cdot 10^{-10}$  (Sandage, 1988).

The measurements of  $^3\text{He}$  abundance are restricted, substantially, to the solar system only. Measurements in the galactic HII regions are very few and are very hard to perform. Additionally, the situation is more complex because during the galactic evolution there is a competition between destruction, survival and production of  $^3\text{He}$ . Therefore, the value of  $^3\text{He}$  as a probe of primordial nucleosynthesis is unclear, although some models of galactic evolution seem to confirm that  $^3\text{He}$  and  $D + ^3\text{He}$  remain close to the primordial values. (Steigman & Tosi, 1992). The observed data seem to imply a value of  $(D + ^3\text{He})/H \sim 4 \cdot 10^{-5}$ . This value is hard to reconcile with the high value of  $D/H$ , found at high redshift. This contradiction can show how the value of primordial abundances (as in the case of  $^3\text{He}$ ), inferred from a galaxy evolution model, can be inadequate. Using these bounds, it is possible to investigate the consistency of the big bang nucleosynthesis model and derive constraints on the nucleon-to-baryon ratio and so on the baryon density  $\Omega_b$ . At the present, a clear result is that the inferred  $\Omega_b$  is much lower than the critical one and the recent data on the high redshift deuterium implied a lower density, barely more than the baryons density we know to be present in luminous stars or high-redshift quasar absorber.

## 4.2 Crucial cosmological tests

The main task of observational cosmology is to select a cosmological theory which is consistent with observational data. From the preceding consideration of the parametric tests it is clear that the possibility exists always to add some new parameters in any cosmological theory and hence to explain a special behavior of the observational data. That is why we have to pay special attention on the crucial cosmological tests which deal with the initial hypotheses of the competing cosmological theories. According to our classification of cosmological theories (see *Fig.1*) we will consider as crucial the following subjects: 1) experimental basis of the gravity theory; 2) cosmological redshift nature; 3) large-scale matter distribution; 4) CMBR nature.

### 4.2.1 Experimental testing relativistic gravity

Experiments in our Solar System can test relativistic gravity only in the weak-field limit, i.e. the post-Newtonian (PN) relativistic effects. So any given relativistic theory of gravity which predicts the correct values of the experimentally measured PN effects could be considered as a possible



future gravitation theory. Such a theory can deviate from General Relativity (GR) in the strong gravitational field and lead to an alternative cosmological model. GR is a geometrical theory of gravitation which describes gravity as a property of space-time. GR predictions such as the perihelion advances of a planetary orbit, the bending and delay of light rays passing near the Sun, and the gravitational redshift of spectral lines had been experimentally checked in the Solar System with an accuracy of about one percent and it was a great success of that theory. The recently discovered pulsars in gravitationally bound binary orbits, provide new astrophysical laboratories for testing gravity at much more stronger fields. In the case of the binary pulsar PSR1913+16 the GR prediction for the combined effect of the gravitational redshift and the special relativistic time dilation has been measured with an accuracy better than 0.07% and the advance of the periastron with an accuracy of 0.0004% (Taylor et al., 1992). Moreover the new relativistic effect-energy loss via gravitational radiation was discovered. According to Taylor et al. (1992) the observed energy loss exceeds the theoretical prediction for the quadrupole gravitational radiation of 0.96% with an accuracy 0.4%. It has been shown by Damour & Taylor (1991) that one must take into account the effect of the galactic rotation and the proper motion of the pulsar. The distance to PSR1913+16 is a very sensitive parameter in the calculation of the galactic effect. If the distance is in the interval  $3 - 8 kpc$  the galactic effect is 0.11% - 0.69% respectively, so that the problem of the excess of the orbital energy loss needs further study.

Tensor field gravitation theory (TFT) describes gravity as a physical interaction caused by exchange of gravitons in flat space-time. All the post-Newtonian effects in the Solar System and in the binary pulsars are the same as in GR, but the interpretation of the effects differs. For example the 16.7% of the periastron advance is due to the positive energy density of the gravitational field distributed around the gravitating neutron stars. Hence the energy density of the static gravitational field has been measured with an accuracy better than 0.002%. Within the TFT there are tensor quadrupole and scalar monopole gravitational radiations. In the case of the binary pulsar PSR1913+16 the monopole radiation provides 0.735% energy loss excess (Baryshev, 1994b). Within the scope of the TFT there is a new interpretation of the Planck mass and a possibility exists for quantum gravity effects in a weak gravitational field (Baryshev & Raikov, 1994) and hence for testing the quantum nature of the gravitational interaction.

#### 4.2.2 Nature of redshift

In *section 2.2.3* we discussed possible mechanisms of the cosmological redshift. About sixty years ago Hubble & Tolman (1935) suggested several tests to discriminate between different redshift mechanisms, but only now the observational technique have reached an adequate level. One of such tests is the surface brightness versus redshift test. In the cases of space expansion, Doppler and gravitational redshift the surface brightness (SB) of a standard source decreases with increasing redshift as  $(1 + z)^4$ . In the case of tired light the SB should vary with redshift only as  $(1 + z)$ .

Recently Sandage & Perlmutter (1991) considered observational data for the first ranked galaxies in 56 nearby clusters and groups. It was demonstrated that after removing all biases and reducing the SB values to a standard condition, there was a  $(1 + z)^4$  dependence of the SB on redshift. This is a first piece of evidence that the cosmological redshift is caused by space expansion, Doppler, or gravitational effects and that it is not the tired light effect. However if one introduces an evolution of the SB within the scope of Tired Light cosmology it is possible to incorporate the result in the scheme.

Another test of the redshift nature was proposed by Sandage (1962) and is related to observations of the redshift of the same object at different epoch, i.e.  $z(t)$  dependence. If the Universe expands with a certain deceleration parameter  $q_0$  the redshift of a fixed object will vary with time so that  $(dz/dt)_0 \approx -H_0 q_0 z$ . In terms of frequency variation it will be  $1.7 \cdot 10^{-14} (day)^{-1}$ . This

test can distinguish between Doppler and gravitational effects.

In *section 3.2.5* we have considered the inhomogeneity paradox, i.e. the linearity of the redshift-distance relation deeply inside the fractal-like inhomogeneity cell. The problem is that in the SM the linear Hubble law is a consequence of the homogeneity. So the measurements of distance independent from redshifts and the measurements of the large-scale structure at the same distances could be considered as a test of the redshift nature.

### 4.2.3 Large-scale matter distribution

The existence of very large-scale structures has been raised from many redshift surveys (Tully, 1986, 1987; Paturel et al., 1988, 1994). These structures are limited only by the extent of the survey in which they are detected. The new correlation analysis that we have discussed in *section 3.4* reconciles the statistical studies with the observed LSS. We have extensively discussed in *section 3.4.3* and *section 3.4.4*. the main properties of visible matter distribution and we refer to these sections for an detailed analysis of the experimental data. Here we summarize the main results.

- $D \approx 2$ . The fractal dimension of galaxy distribution approaches to 2 in many independent redshift surveys such as CfA2 (Park et al., 1994), QDOT (Moore et al., 1994), Perseus-Pisces (Guzzo et al., 1992) and ESP (Sylos Labini & Pietronero, 1995a).
- $\lambda_0 \gtrsim R_s$  where  $\lambda_0$  is the scale of homogeneity and  $R_s$  is the depth of the various currently available redshift surveys. No clear cut-off towards homogenization has yet been identified in the available samples ( $R_s \gtrsim 200 - 400 Mpc$ ). In the next few years there will be available many new redshift surveys (see for a review Efstathiou, 1994) that will cover a fraction of the entire Hubble radius. From these data a definitive measurement of the eventual scale of homogeneity will be soon available.
- $D \approx 1.5$  for the clusters distribution, even though there are a lot of uncertainties in the exact value of the fractal dimension due to the poor statistics of cluster catalogues. Recently Borgani et al. (1994) found that the fractal dimension turns out to be  $D \sim 2.2$ . for both Abell and ACO clusters.
- The galaxy luminosity function is well described by a Schechter function.
- Low-surface-brightness and dwarf galaxies seem to fall into the structures delineated by the luminous ones and there is no evidence that these galaxies fill voids (see *section 2.4*).
- The whole mass distribution can be investigated by its gravitational effects studying the large-scale bulk flows (*section 4.1.5.2*.) and the gravitational lensing (*section 4.1.5.3*.). The full extent of these flows is still unknown and it is not clear if the dark matter is traced by the luminous one. The gravitational lensing is a new tool that is at the beginning of its development.

### 4.2.4 Cosmic Microwave Background Radiation

There are several kind of observations that have been done on the CMBR from its discovery in 1968 until now (Penzias & Wilson, 1965; see for a review Melchiorri & Melchiorri, 1994). The absolute radiometry intends to determine the temperature and the spectrum of the CMBR. The most accurate spectrum is obtained with the observation of a region of low dust density content

and it was found by the COBE team (Mather et al., 1990, 1994) and independently by Gush (1990) that it is a perfect black body spectrum with a temperature:

$$T = 2.726 \pm 0.010 K \quad (4.2)$$

This is the most accurate determination of the CMBR in the millimetric and submillimetric region ( $1 - 20 \text{ cm}^{-1}$ ). Deviations from this blackbody are less than 1 % of the peak brightness. The situation is more complex at higher wavelengths (Melchiorri & Melchiorri, 1994). The CMBR is remarkably close to isotropic, but there is a clear indication of an existence of a dipole anisotropy. COBE (Fixsen et al., 1994) has provided both the spectrum and the direction of dipole anisotropy with high accuracy. The observed temperature difference between two regions in the sky, which lie in the opposite directions, is  $\Delta T = 3.343 \pm 0.016 \text{ mK}$ . If this anisotropy is due to the motion of our galaxy with respect to the CMBR this is an important cosmological tool when compared with the observed peculiar velocities of other galaxies (see *section 4.1.5.2*).

The detection of small as well as large-scale anisotropies is a very difficult task (Melchiorri & Melchiorri, 1994). The main problem is to identify the possible source of spurious signals that can be instrumental and local environmental effects, atmospheric, Solar System, galactic and extragalactic disturbances. Few groups have detected CMBR anisotropies at various angular scales while many other have obtained upper limits only. The amplitude of these fluctuations, after having taken into account the various spurious contributions, is of the order of some  $\Delta T/T \sim 10^{-5}$ . Hence it is very hard to decide if the signals observed by Relict 1 (Strukov et al., 1992), by COBE (Smoot et al., 1992) and other groups are CMBR anisotropies or a mixture of CMBR and spurious signals. These results pose some trouble for the SM. In fact in the standard scenario of galaxy formation, small amplitude primordial perturbations to the energy density are amplified by gravitational instability as the Universe expands and they are predicted to leave a detectable imprint on the CMBR. Such low amplitude measured fluctuations are not compatible with the standard baryonic matter and hence in many theories of galaxy formation one must introduce some exotic kind of non baryonic matter that has a weaker interaction with photons than the baryonic one. Moreover on large angular scale (few degrees) the dominant mechanism by which density fluctuations induce anisotropy in the CMBR is the Sachs-Wolf effect (Sachs & Wolf, 1967). This is a gravitational effect due to the presence of matter between the CMBR and the observer. For this reason the existence of large-scale structures can represent a serious problem due to their incompatibility with such low amplitude temperature fluctuations.

According to the Hot Big Bang (HBB) model the temperature of the CMBR must scale in proportion to  $(1 + z)$ , where  $z$  is the redshift at which the radiation is measured. Clearly such a measurement is a crucial one because it is a direct proof in favor (or not) of the HBB scenario. This effect can be tested by measuring the populations of excited fine-structure lines in the absorption spectra of distant quasars. Observations of this kind is strongly limited by the signal-to-noise ratio of the available telescopes (Meyer et al., 1986). Recently (Songaila et al., 1994a, 1994b) found a meaningful  $2\sigma$  upper limit of  $T = 13.5 K$  for the temperature of the CMBR in a cloud that contains singly ionized carbon at  $z = 2.9$ , and have measured the excitation temperature of  $7.4 \pm 0.8$  for the first fine-structure level of neutral carbonic atoms in a cloud at redshift 1.776. These upper limit are very close to the temperature expected on the basis of the HBB scenario that is  $T = 10.7 K$  and  $T = 7.58 K$ , of course assuming that no other significant sources of excitation (collisions, radiation) are present. It is to be hoped that more definitive measurements will be soon available because a firm measurement of the CMBR temperature at high redshift would permit to rule out or not the HBB theory.

## 5 DISCUSSION AND CONCLUSIONS

Cosmology as an experimental science must be based on experimentally checked hypotheses, and the main task of this paper is to analyze what are facts and what are only ideas in contemporary cosmology. To do this, one requires a comparison of cosmological models with observational data and we have divided (*sec. 2.1.,2.2*) the possible experimental tests into two different kinds. The first ones are the crucial tests and deal with the fundamental basis of any cosmological theory. They check the validity of the initial assumptions and hypotheses of various theories. The second kind of tests are the parametric ones and give estimates of the parameters of different models. The whole set of parametric tests can play the role of a crucial one if no other free parameters of the model are available. We have classified different cosmological theories (Fig.1.) according to their answer to the following questions: what is gravity, how matter is distributed in space, what is the nature of the redshift, what is the nature of the CMBR and what is evolution and the arrow of time. Therefore according to this classification, the crucial tests deal with these fundamental matters: gravitation, matter distribution, redshift nature, and Cosmic Microwave Background Radiation (CMBR) nature.

Experiments on gravitation are performed only in weak field approximation. In this limit General Relativity (GR) gives predictions that are in good agreement with the results of various experiments. We discuss (*sec. 2.2.1.*) different approaches to the gravitation theory that give basically the same results of GR in a weak field, but that can be different in stronger fields. Among these approaches we have discussed in particular the tensor field theory in flat space-time that can represent an alternative direction with respect to the geometric theories.

The matter distribution (*sec. 2.2.2.*) in space represents the most powerful test of the basic initial hypothesis of the main cosmological theories: the Standard Friedman Model (SM) and the Steady-State Model (SSM). In fact, both theories assume, beyond a certain scale  $\lambda_0$ , the homogeneity of matter distribution. The initial hypothesis of homogeneity has been justified as required by the *Cosmological Principle* (CP) in order to avoid any privileged observer in the Universe. In our opinion, nowadays, the main experimental problem is the value of  $\lambda_0$ . We have discussed in detail this point stressing that from many redshift surveys there is no clear evidence towards homogenization, but on the contrary a well defined fractal behavior is found. Moreover the condition of local isotropy, that is satisfied in a fractal structure, is the necessary condition in order to ensure the statistical equivalence of all the observers. Hence the CP can be saved in a fractal distribution in a more weaker version, i.e. without the strong request of the complete transitional and rotational invariance (homogeneity).

Another initial hypothesis of the SM and SSM is the *Expanding Space Paradigm*. According to this assumption the redshift and the linearity of the Hubble law are due to space expansion. Actually (*sec. 2.2.3.*) there are four different kinds of physical mechanisms that can produce a redshift: the Doppler, the gravitational, the space expansion and the tired light effects. Of these, only the first two are tested in laboratory experiments, while the space-expansion and the tired light redshifts have never been experimentally proved. We have discussed in detail the gravitational redshift in a homogeneous and in a fractal structure starting from the spherically symmetrical Bondi-Tolman model. We have stressed that for a fractal with dimension  $D = 2$ , it is possible to obtain a linear  $z$ - $d$  relation, whose amplitude is related to the lower cut-offs of the fractal distribution. We have particularly emphasized that the observational quantity is the redshift and not the expansion velocity.

The CMBR (*sec. 2.2.4.*) together with the three dimensional space distribution of galaxies is the most important fact of modern cosmology. The main characteristics are its present temperature, its nearly perfect blackbody spectrum, its extraordinary isotropy and its energy content that is of the same order of magnitude as local sources. The extragalactic nature of the CMBR

is without doubts for the observed transparency of the Universe up to the distance of Quasars, but there is not clear experimental evidence, but only upper limits, that its temperature scales linearly with redshift, as predicted by the Hot Big Bang (HBB) scenario.

The discussion on modern cosmological ideas begins (*sec. 3.1.*) from the well known paradoxes of Newtonian cosmology such as the gravitational, the Olber's and the thermodynamics paradoxes. These paradoxes pose fundamental problems. To resolve a paradox one needs to consider the initial postulates of the model used.

The SM (*sec. 3.2.*) is based on the following hypothesis: the GR is the correct gravity theory, the density of matter is constant (at fixed time) so that the Universe is homogenous beyond a certain scale  $\lambda_0$ , the redshift and the linearity of the Hubble law ( $z - d$  relation) are due to space expansion and, finally the laws of thermodynamics hold in the expanding space. The main success (*sec. 3.2.2.*) of the SM lies in the prediction of the blackbody spectrum of the CMBR, while it does not determine its temperature and does not explain its isotropy. Another great success of the SM is the predictions of light elements abundances that are in good accordance with observational constraints. The crucial tests (*sec. 3.2.3.*) of the SM concern the GR in a strong field, the detection of the homogeneity scale  $\lambda_0$  of matter distribution, the reality of space expansion and then that of the expansion-redshift, the experimental verification of the relation  $T(z) = T_0(1 + z)$ , the comparison of the age of the Universe with the age of the oldest objects and finally the comparison of theoretical predictions with more stringent data on chemical compositions. The classical parametric tests that we have considered (*sec. 3.2.4.*) for a comparison with experimental data are the redshift-distance, redshift-angular size, redshift-magnitude, number-magnitude, number-redshift and time-redshift relations. There are 5 paradoxes of the SM (*sec. 3.2.5.*) of which 4 are well-known and the last one has only recently been discussed. In fact the flatness, the isotropy, the superluminal velocity and the global energy paradoxes are widely discussed in the literature and there are attempts to resolve some of these paradoxes in the framework of Big Bang models. The so-called *inhomogeneity paradox* is the following: if the Hubble law is a consequence of homogeneity, then there is a contradiction of the nearly linear  $z - d$  relation at the same scales (at least  $2 - 20\text{Mpc}$ ) where one observes well defined fractal behavior and large-scale departures from homogeneity. It is possible to characterize quantitatively the expected behavior of the  $z - d$  relation in the case of fractal distributions of matter: the result is that inside the "inhomogeneity-cell", i.e. for distances  $\lesssim \lambda_0$  a strong non-linearity is expected. To resolve this paradox one should hypothesize that the dark matter is homogenous at very small scales (of order of some Mpc) and hence one introduces a very large amount of dark matter that is in conflict with the dynamical estimates in the galactic halos and galaxy clusters.

To resolve some of the problems of the SM non standard models have been introduced (*sec. 3.2.6.*) that are the  $\Lambda - \text{term}$  model (*sec. 3.2.6.1.*) and the inflationary Universe. In both these models one avoids some particular problems of the SM but introduces some new hypothesis that has to be checked experimentally. In the case of the  $\Lambda - \text{term}$  model one can make the age of the Universe larger than in the SM (for a fixed density) and modify the geometry of the Universe, so that a flat space is allowed also with a density parameter lower than 1 but with  $\Omega + \Lambda = 1$ . The problem, here, is to explain the particular value of the cosmological constant used, i.e. the famous *cosmological constant problem*: The theoretical expectation value for the cosmological constant exceeds the observational limits by some 120 orders of magnitude. In the inflationary scenario (*sec. 3.2.6.2.*) one introduces a phase of exponential expansion during the evolution of the Universe that can explain the flatness and the isotropy paradoxes. The scalar field, that is central in this scenario, comes from the Grand Unified Theories (GUT) of weak and strong interactions. The problems in this approach are the strong initial assumptions and the poor experimental verifications of GUT theories. In any case this model does not resolve the other paradoxes of the SM.

In the Steady-State Model (SSM) (*sec. 3.3. and 3.3.1.*) it is assumed that the density of matter is constant in space and in time (Perfect Cosmological Principle), that a modified version of GR is the gravity theory and that the redshift is due to the expansion of space. There are predictions (*sec. 3.3.2.*) for the classical parametric tests that, at the first order in  $z$ , do not differ from the SM's ones. The flatness and the isotropy paradoxes are avoided, but there are still the superluminal, the global energy and the inhomogeneity paradoxes. In the new version of SSM, the "Quasi- Steady State Model" (*sec. 3.3.3.*) there is attempt to explain the CMBR as a post-stellar scattered radiation thermalized by a special kind of intergalactic medium. The thermalising process can be one of the crucial aspects of this model, together with the crucial tests that we have discussed for the SM.

A fractal model (*sec. 3.4.* has not been developed yet. Hence, first of all we have focused (*sec. 3.4.1.*) on the main properties of non-analytic and self-similar distributions stressing which are the new concepts that can have an important impact not only in theoretical models but also in data analysis. In fact fractals are intrinsically non-analytical distributions characterized by having self-similar structures and voids at all scales (*sec. 3.4.2.*). For this kind of distributions, the correlations properties are described by power law functions. The main point is that one cannot discuss self-similar structures in terms of amplitude of correlations, neither in a theoretical model nor in the data analysis. The only physically meaningful property is the exponent of the correlation function, and this is the crucial quantity that a theoretical model needs to explain. The amplitude of the correlation function, as well as other related quantities such as the normalized density fluctuations ( $\delta N/N$ ), are spurious. We have stressed therefore that for a self-similar structure it has no physical meaning to discuss the dynamics in terms of "linear" and "non-linear" regimes, that are identified by the scale at which the fluctuations become negligible with respect to the average (i.e.  $\delta N \sim N$ ) because the average itself is not well defined, because it depends from the sample size. Hence the concept of "big" and "small" amplitudes are misleading for a self-similar distribution, because is not defined a constant average density. We have discussed in detail (*sec. 3.4.2.*) the analysis *without assumptions* that we have performed on real redshift survey, such as *CfA1*, *Abell*, *CfA2*, *ESP*. From this discussion it is clear that the new redshift surveys data imply a deep change of the theories of galaxy formation, such as the biased galaxy formation: the clusters correlations are just the continuation of the galaxy correlations at larger scales and the galaxy-cluster mismatch, i.e. the different amplitude of correlations function for galaxies and clusters, is simply solved from this point of view.

If one considers the whole mass distribution, the concept of multifractal (MF) naturally arises (*sec. 3.4.4.*) that is just a refinement of the concept of fractal and, indeed, it is not in contrast with it. Hence if the whole mass distribution is self-similar the space locations of galaxies are correlated with their luminosities, and this correlation can be studied and quantitatively characterized with the MF formalism. Analyzing the properties of fractal (and MF) distributions in the three dimensional Euclidean space, one can make predictions for some of the classical parametric tests, such as the number-counts and the redshift-magnitude relations. We find that in this case, at small redshifts, there is a good agreement with the available data. Actually the situation at intermediate and large redshifts is not clear because of selection effects in the data, in the data analysis and the possible evolutionary effects that can be relevant at these distances. The problem here lies in the fact that if one defines evolution as the difference between theoretical predictions and experimental data, one should be sure that the model has been widely verified. The correlation between space locations and masses of galaxies is the important element that should be taken into account not only in theoretical models, but also for the data analysis. The luminosity function (LF) of galaxies can be naturally related to the MF properties and in particular its exponent can be related to the MF spectrum. The amplitude of the LF is related to the average density and hence it is not constant but depends upon the sample depth. These considerations have

an important impact also in the experimental methods for determining the luminosity function. Moreover we have described a simple stochastic model based on the aggregation of particles. The main aim of such a model is to study which are the characteristics of the dynamical process that can give rise to a fractal, and a MF if one includes masses distribution. We have identified in the breaking of the spatial symmetry of the aggregation process and the Self-Organized nature of the dynamical mechanism, the key elements in order to generate such structures.

There have been some interesting attempts towards the so-called "Newtonian fractal models" (*sec. 3.4.5.*) in particular because they permit us to solve two famous paradoxes, the gravitational and the Olbers ones, without need to modify the Newtonian gravity theory. In the framework of General Relativity (GR) there have been some attempts to describe fractal distributions (*sec. 3.4.6.*). In particular using the inhomogeneous spherical symmetric metric (the Bondi-Tolman metric) joined with a Friedmann metric (i.e. the so-called *Swiss cheese model*) one avoids any privileged observer and saves the equivalence of all points. In this model it is possible to find solutions that agree with two main experimental facts: the linear redshift-distance relation at small redshift and the fractal behavior along the backward null cone with the right experimental values of the Hubble constant and the fractal dimension. Therefore it is possible to describe such complex structures in GR even if this model does not provide an answer to the origin problem. But this is the first methodological example of including fractal models in the framework of GR. Moreover it clarifies the point that in GR it is possible to describe systems with a vanishing average density: the key-point is that the average density is made not at a space-like hypersurface of constant time, but such averaging is carried out along the backward null cone.

We considered also a possibility of the construction of a cosmological model based on the tensor field gravitation theory (TFT) and fractal distribution. In particular, we note that TFT, with the additional condition of the positiveness of the energy density of the gravitational field, substantially differs from GR in the case of strong fields and infinite matter distribution. In weak fields approximations, it gives the same predictions of GR. As an example, we construct a static cosmological model, with cosmological redshift due to the gravitation of the fractal matter distribution with fractal dimension  $D = 2$  *sec. 3.4.7.*

There are several other alternative cosmologies, which have been discussed in the literature. Usually there is no a definitive formulation of the initial hypotheses of the models. Further development of the models to make it possible a comparison of its predictions with observations is needed (*sec. 3.5*).

According to our classification of observational data, parametric tests permit the determination of the various parameters of a model. The whole set of parametric tests can be considered as a crucial one if no other free parameter can be added to the model (*sec. 4.1.*).

Up to now the SM only has definite predictions for classical parametric tests. The most important parametric test is the redshift-distance relation (*sec. 4.1.1.*). The main results are the linearity of the  $z - d$  Hubble-law in the range  $\sim 2 - 100 Mpc$  and the value of the Hubble constant that, from various independent measurements, seems to converge now to the value of  $\sim 80 kmsec^{-1} Mpc^{-1}$ . This high value may represent a problem for the SM unless one does allow the density parameter less than one. Moreover the linearity of this relation is difficult to reconcile, if due to space expansion, with a highly inhomogeneous distribution of matter at the same scales. This is the *inhomogeneity-paradox* that we have previously discussed. On the contrary if we consider the cosmological redshift as due to the gravitational redshift effect, without space expansion, one obtains a linear  $z - d$  relation in highly inhomogeneous matter distribution. On the angular size-distance relation there are different results according to the different "standard-sticks" used (*sec. 4.1.2.*). The experimental data are therefore problematic and no definitive answer on the geometry of the Universe is accepted. The slope of the magnitude-redshift ( $< m > \sim \log(z)$ ) relation has been found to be 5 with fair accuracy (*sec. 4.1.3.*). This result does not permit

us to discriminate between a fractal and an homogenous distribution. In fact, we have shown that both distributions at low redshift (Euclidean space) have the same behavior. Moreover there is a discrepancy between the results in the B-band and the K-band. At larger redshifts the experimental situation is highly uncertain and an evolutionary model has probably to be taken into account. The problem of which particular evolutionary model one chooses, as discussed before, is very difficult. The exponent of the galaxy counts ( $\log(N < m) \sim \alpha m$ ) is connected with the fractal dimension of matter distribution only if it comes from the analysis of the number-counts in volume limited samples and under the assumption that there is no correlation between space locations and luminosities of galaxies. We have discussed in detail these effects that have never been taken into account either in the analysis, nor in the theoretical models (*sec. 4.1.4.*). Even in this case no definitive answers on the value of the fractal dimension and on the geometry of the space are available. Moreover the evolutionary, the bias and selection effects can play an important role, and can explain the different slopes measured in different frequency-bands.

The age of the oldest objects is a very stringent test for the validity of SM models (*sec. 4.1.5.1.*). In fact typical estimates of globular cluster ages and radioactive decay ages give respectively  $T_0 \sim 14 - 16 Gyr$  and  $T_0 \sim 9 - 16 Gyr$ . In the SM these ages are in conflict with a high value of the Hubble constant ( $H_0 \gtrsim 75 kmsec^{-1} Mpc^{-1}$ ) unless one introduces a cosmological constant different from zero.

To study the whole mass distribution, including the dark matter, it is important to consider the bulk flows, i.e. the net peculiar velocity of galaxies averaged over big volumes (*sec. 4.1.5.2.*). The main results that in these last years have been obtained from many independent surveys, is that the full extent of these flows is still uncertain and it is limited only by the sample size. In fact very large-scale coherent flows have been detected, over scales of  $\sim 150 Mpc$ . It is very hard to reconcile these results with an homogeneous picture, where the mass fluctuations have to be small on large scales. In a fractal distribution the fluctuations are intrinsic and the extent of these flows can be limited only by the scale of isotropy  $\lambda_i$  that has not been clearly detected in the available samples.

A new tool to study the whole mass distribution is represented by the gravitational lensing (*sec. 4.1.5.3.*). We have shown the difference between an homogenous and a fractal distributions of lenses. The experimental data are still scarce, but in the near future this may represent a powerful test to study matter distribution on very large scales.

The light elements abundances estimations are affected by the chemical evolution of the galaxy (*sec. 4.1.5.4.*). Hence the inferred primordial abundances are strongly model dependent results. Only recently the estimates of extragalactic abundances such as deuterium are available. In any case the predicted abundance of helium and deuterium are well in agreement with the predictions of the standard Hot Big-Bang nucleosynthesis. Also the revised version of the SSM is claimed to reproduce the observed abundances.

As we stressed previously experiments on gravitation were performed only in weak field approximation. The recently discovered pulsars in gravitationally bound binary orbits provide a new laboratory for testing predictions of GR as of TFT (*sec. 4.2.1.*). In fact, in the case of the binary pulsars PSR 1913+16, it is predicted, within the TFT, the existence of 0.735% excess of gravitational radiation, due to the scalar gravitation waves. It is very important for cosmology that in the near future, observational technology will reach the necessary level for testing of the redshift nature. Recent observations of the surface brightness of giant elliptical galaxies (Sandage & Perlmutter, 1991) showed a first experimental indication against tired light (TL) mechanism of cosmological redshift. However, these observations can not discriminate between space expansion, Doppler and gravitational effects (*sec. 4.2.2.*).

Among the crucial tests, the investigations of the large-scale matter distribution has been strongly developed in the last decade (*sec. 4.2.3.*). Now there are now available three dimensional



redshift surveys that cover a large fraction of the Hubble radius. Moreover in the next few years new and larger redshift surveys will be completed. From these catalogues emerges the new picture of the Universe that is dominated by large-scale structures and voids of all scales. In fact, the extent of the largest structures and voids recently discovered is limited only by the boundaries of the surveys. The homogeneity scale  $\lambda_0$  has not been identified and seems to shift to a very large value, at least 10 times greater than the previously believed scale inferred from the angular catalogues ( $\sim 5Mpc$ ). This new situation clearly poses new crucial problems to the classical paradigm. The fractal dimension of the visible matter distribution converges to the value  $D \approx 2$  from many independent redshift surveys. The galaxy cluster distribution shows power law correlations that are the continuation of the galaxy correlations at more deeper scales. Hence clusters and galaxies are part of the same self-similar and scale-invariant distribution. A very important observational cosmology, in our opinion, is the determination of the eventual homogeneity scale  $\lambda_0$  and this is a crucial point because the assumption of homogeneity is the cornerstone of the SM and SSM.

The Cosmic Microwave Background Radiation represents the other well studied observational fact in modern cosmology (*sec. 4.2.4.*). Its temperature and spectrum has been determined very accurately. The isotropy of the CMBR blackbody spectrum has been intensively studied. The result is that the anisotropies are probably detected at  $\Delta T/T \sim 10^{-5}$ . The extragalactic origin of this radiation is without doubt. The crucial test for the Hot Big-Bang scenario is the measurements of the CMBR temperature at large redshift, and in particular the experimental proof that the temperature scales as  $(1+z)$ . Actually only upper limits are available, but with the use of new large telescopes in the following years it is hoped to obtain more stringent results.

Our conclusion is that some of the basic assumptions of the SM and SSM are in conflict with the new observations and tests. It is important therefore to focus the attention of the experimental crucial tests that can give a definitive answer to the validity of the classical paradigm. Moreover it should be stressed that the fractal behavior of matter has changed our view of galaxy correlations that are now compatible with the observed large-scale structures. This is the new and fundamental element that has to be taken into account in any cosmological model. The significant growth of the observational data in cosmology leads to a new situation, because the Standard Model has to use many ad hoc parameters to explain observations. But such procedures can be made within other alternative models and the exceptional role of the Standard Model disappear. That is why now it is a good time for more careful analysis of initial hypotheses of different cosmological theories. To distinguish between alternative possibilities one must develop the crucial observational tests which allow us to check the initial postulates of the models. We emphasized that such tests will be fulfilled in the near future.

### *Acknowledgments*

We are particularly grateful to Dr. P. Vettolani, for giving us the possibility of analyzing the ESP survey and to Dr. P. Coleman for continuous discussions and collaborations. We thank for useful discussions, criticisms and suggestions L. Amendola, P. Bak, R. Cafiero, R. Capuzzo-Dolcetta, M. Costantini, P. de Bernardis, P. Grassberger, M. Geller, R. Kolb, A. Linde, V. Loreto, B. Mandelbrot, F. Melchiorri, S.A. Oschepkov, G. Paladin, A.A. Raikov, R. Scaramella, D. Schramm, M. Ribeiro, A.G. Sergeev, G. Setti, P. Teerikorpi, D. Trevese, S.A. Tron', N. Vittorio, G. Zamorani.

## 6 References

### References

- [1] Abell, G. O. (1958) The distribution of rich clusters of galaxies. *Astrophys. J. Suppl.*, **3**, 211
- [2] Alfven, H. (1989) Cosmology in the plasma universe. *Laser Part. Beams* **6**, 639-98.
- [3] Arp, H. C., Burbidge, G., Hoyle, F., Narlikar, J. V., Wickramasinghe, N. C., (1990) The extragalactic Universe: an alternative view. *Nature* **346**, 807-812.
- [4] Babul, A, Rees, M, (1992) On dwarf elliptical galaxies and the faint blue counts. *Mon.Not.R.astr.Soc.***255**,346-354
- [5] Bahcall, N. A. (1986) The difference between the galaxy and cluster correlation functions: a manifestation of tails of galaxy clusters. *Astrophys. J. Lett.* **302**, L41-L44.
- [6] Bahcall, N. A., Batuski, D. J.(1988) The correlation function of southern clusters. *Astrophys. J.* **333**, L13-17
- [7] Baryshev, Yu. V. (1981) Hierarchic structure of the Metagalaxy: *Izvestiya SAO* **14**, 24 - 43 (English translation: 1984 by Allerton Press, Inc.)
- [8] Baryshev, Yu. V., (1988) Conservation laws and equations of motion in the field gravitation theory, *Vest. Leningrad. Univ. ser.1, vyp.2*, 80-85.
- [9] Baryshev, Yu. V., Kovalevski M.A. (1990) Homogeneous ball in the field gravitation theory, *Vestn. Leningrad. Univ., ser.1, vyp.1* 86-91.
- [10] Baryshev, Yu. V. (1994a) On the fractal nature of the Large Scale Structure of the Universe. *Astron. Astrophys. Trans.* **5**, 15-23.
- [11] Baryshev, Yu. V., (1994b) On a possibility of scalar gravitational wave detection from the binary pulsar PSR1913+16, in the Proc. of the *First Amaldi Conference on Gravitational Wave Experiments*, in print.
- [12] Baryshev, Yu. V., Raikov A.A. (1994) A quantum limitation on the gravitational interaction, in the Proc. of the Intern. Conf. *Problems on High Energy Physics and Field Theory*, in press.
- [13] Baryshev, Yu. V. (1995) Relativistic gravity in flat space-time, in preparation.
- [14] Baryshev, Yu. V., Pietronero, L., Sylos Labini, F.(1995) Gravitational lensing in a fractal distribution, submitted to *Europhys.Lett*
- [15] Baryshev, Yu. V., Teerikorpi, P. (1995) Kinematical models of double radio sources and the unified scheme, *Astron. Astrophys.*, in press
- [16] Benzi,R., Paladin, G., Parisi, G.,Vulpiani, A. (1984) On the multifractal nature of fully developed turbulence and chaotic systems. *J.Phys. A.* **17**, 3251-3531.
- [17] Binggeli, B., Sandage, A., Tamman, G. A. (1988a) The luminosity function of galaxies. *Astron. Astrophys. Rev.* **26**, 509
- [18] Binggeli, B., Sandage, A., Tamman, G. A. (1988b) Studies of the Virgo Cluster. VI. Morphological and kinematic structure of the Virgo Cluster. *Astron. J.* **94**, 251-77.
- [19] Binggeli, B., Tarenghi, M. Sandage, A., (1990) The abundance and morphological segregation of dwarf galaxies in the field. *Astron.Astrophys.*, **228**, 42
- [20] Blandford, R. D., Narayan, R. (1992) Cosmological applications of gravitational lensing. *Ann. Rev. Astron. Astrophys.* **30**, 311-358.
- [21] Blau, S. K., Guth, A. H. (1987) Inflationary cosmology, in *300 years of Gravitation*, Ed. by Hawking, S. W. & Israel,W.,pp.524-603 Cambridge Univ. Press, Cambridge.

- [22] Boesgaard, A. M., Steigman, G. (1985) Big Bang nucleosynthesis: theories and observations. *Ann. Rev. Astron. Astrophys.* **23**, 319-378.
- [23] Bogolubov, N., Shirkov, D. (1976) Introduction to quantum field theory, Nauka, Moscow
- [24] Bondi, H. (1947) Spherically symmetrical models in general relativity. *Monthly Not. Roy. Astron. Soc.* **107**, 410-425.
- [25] Bondi, H., Gold, T. (1948) The Steady - State theory of the expanding Universe. *Mon. Not. R. Astr. Soc.* **108**, 252-270.
- [26] Borgani, S., Martinez, V.J., Perez, M.A., Valdarnini, R. Is there any scale in the clusters distribution ? *Astrophys. J.*, **435**, 37.
- [27] Bowler, M. (1976) Gravitation and Relativity. Pergamon Press, N.Y.
- [28] Brans, C., Dicke, R. H. (1961) Mach's Principle and a relativistic theory of gravitation *Phys. Rev.*, **124**, 925.
- [29] Broadhurst, T. J., Ellis, R. S., Koo, D. C., Szalay, A. S. (1990) Large-scale distribution of galaxies at the Galactic poles. *Nature* **343**, 726-728.
- [30] Broadhurst, T. J., Ellis, R. S., Glazebrook, K. (1990) Faint galaxies: evolution and cosmological curvature *Nature* **355**, 55-58
- [31] Bronstein, M. D. (1934) On the possible theory of the world as a whole. In *Main problem of cosmic physics* 186-215 ONT I, Kiev.
- [32] Calzetti, D., Giavalisco, M., Ruffini, R. (1988) The normalization of the correlation functions for extragalactic structures. *Astron. Astrophys.* **198**, 1-15.
- [33] Carroll, S. M., Press, W. H., Turner, E. L. (1992) The cosmological constant. *Ann. Rev. Astron. Astrophys.* **30**, 499-542.
- [34] Cavalleri, L., Spinelli, F. (1980) Field theoretic approach to gravity in the flat space-time. *Rev. Nuovo Cimento* **3**, 1-92
- [35] Chambers, K.C., Miley, G.K., Breugel, W.J.M. (1990) 4C41.17: A radio galaxy at a redshift of 3.8 *Astrophys. J.* **363**, 21-39.
- [36] Chandrasekhar, S. (1943a) Dynamical friction I. general considerations: the coefficient of dynamical friction. II. The rate of escape of stars from cluster and the evidence for the operation of dynamical friction. *Astrophys. J.* **97**, 255-273.
- [37] Chandrasekhar, S., Neumann, J. Von, (1943b) The statistics of the gravitational field arising from a random distribution of stars. II. The speed of fluctuations; dynamical friction: spatial correlation. *Astrophys. J.* **97**, 27.
- [38] Chandrasekhar, S. (1943c) Dynamical friction III. A more exact theory of the rate of escape of stars from clusters. *Astrophys. J.* **98**, 54-60.
- [39] Chandrasekhar, S. (1943d) Stochastic problems in physics and astronomy. *Rev. Mod. Phys.* **15**, 89.
- [40] Charlier, C. V. L. (1908) Wie eine unendliche Welt aufgebaut sein kann. *Arkiv for Mat. Astron. Physik* **4**, 1.
- [41] Charlier, C. V. L. (1922) How an infinite world may be built up. *Arkiv for Mat. Astron. Physik* **16**, 1-35.
- [42] Coleman, P. H., Pietronero, L., Sanders, R. H. (1988) Absence of any characteristic correlation length in the CfA catalogue. *Astron. Astrophys.* **245**, 1.
- [43] Coleman, P. H., Pietronero, L. (1992) The fractal structure of the universe. *Phys. Rep.* **213**, 311-391.
- [44] Coles, P., Ellis, G. (1994) The case for an open Universe. *Nature* **370**, 609-615.

- [45] Colless, M., Ellis, R.S., Taylor,K.,Hook,R.N (1990) the LDSS deep redshift surveys. *Mon.Not.R.astr.Soc*, **224**,408-423
- [46] Colless, The Abell clusters inertial frame, *preprint*
- [47] Courteau, S., Faber,S.M., Dressler,A., Willick,J.A. (1993) Streaming motions in the Local Universe: evidence for large-scale low-amplitude density fluctuations. *Astrophys.J.Lett.*,**412**, L51-L54
- [48] Croswell, G.,(1993) The quest for cosmological constant. *New Scientist* **20 febr.**, 23-27.
- [49] Da Costa , L.N et al. (1988) The Southern Sky Redshift Survey. *Astrophys.J.*, **327**, 544
- [50] Da Costa et al. (1994) A complete southern sky redshift survey. *Astrophys.J.Lett.* **424**, L1-L4.
- [51] Damour, T., Taylor, J.H.I. (1991) On the orbital period of the binary pulsar PSR 1913+16 *Astrophys. J.* **366**, 501-511
- [52] Davis, M., Huchra, J., Latham, D. W.(1982) A survey of galaxy redshifts. II. The large-scale space distribution. *Astrophys. J.* **253**, 423-45.
- [53] Davis, M., Huchra, J., (1982) A survey of galaxy redshifts. III. The density field and induced gravity field. *Astrophys. J.* **254**, 437-50.
- [54] Davis, M., Peebles, P. J. E. (1983) A survey of galaxy redshifts. V. The two-point position and velocity correlation. *Astrophys.J.* **267**,465
- [55] De Lapparent, V., Geller, M. J., Huchra, J. P. (1988) The mean density and the two-point correlation function for the CfA redshift survey slices. *Astrophys. J.* **332**, 44-56
- [56] De Vaucouleurs, G. (1970) The case of a hierarchical cosmology. *Science*, **167**, 1203-1213
- [57] Dirac, P. A. M., (1937) Cosmological constants. *Nature* **139**, 323.
- [58] Disney, M., Phillips,S. (1987) Icebergs and crouching giants. *Nature*, **329**, 203-204
- [59] Djorgovski, S., Spinrad, H. (1981) Towards the application of a metric size function in galactic evolution and cosmology. *Astrophys. J.* **251**, 417.
- [60] Dogterom, M., Pietronero, L. (1991) The angular projection of fractals and its relevance for the galaxy distributions. *Phys. A* **171**, 239-248.
- [61] Dressler, A., (1984) The evolution of galaxies in clusters. *Ann.Rev.Astron.Astrophys.*, **22**, 185-222.
- [62] Dressler, A., Faber, S.M, Burstein,D., Davies R.L., Lynden-Bell, D. (1987) Spectroscopy and photometry of elliptical galaxies: a large-scale streaming motion in the local Universe. *Astrophys.J.*, **313**, 42.
- [63] Dunlop, J.S. et al. (1989) The colour evolution of high redshift radio galaxies. *Mon.Not.R.astr.Soc*, **240**, 257-284.
- [64] Eder, J.A., Schombert, J.M, Dekel, A., Oelmer, A. Jr. (1989), The clustering of dwarf galaxies. *Astrophys.J.*, **340**, 29-46.
- [65] Efstathiou, G., et al. (1991) Large-scale clustering of IRAS galaxies *Mon.Not.R.astr.Soc.* **247**, 10p-14.
- [66] Efstathiou, G., (1994) *Les Houches Lectures 1993* Observations of large-scale structure in the Universe, Elsevier Science Publisher, Netherlands
- [67] Einasto, M., Einasto, J (1987) Structures and formation of superclusters, *Mon.Not.R.astr.Soc*, **226**, 543.
- [68] Einstein, A. (1916) Die Grundlage der allgemeinen Relativitatstheorie. *Am. d. Phys.***49**, 769.
- [69] Einstein, A. (1917) Kosmologische Betrachtungen zur allgemeinen Relativitatstheorie, Sitzungsber. d. Berl. Alead., **1**, s.142

- [70] Fang, L. L., Mo, H. J., Ruffini, R. (1991) The cellular structure of the universe and cosmological tests. *Astron. Astrophys.* **243**, 283-294.
- [71] Fixsen, D.J. et al. (1994) CMB spectrum measured by the COBE FIRAS instrument *Astrophys.J.*, **420**, 445-450
- [72] Fournier d'Albe, E. E. (1907) Two New Worlds. Ed. Longmans Green, London.
- [73] Freedmann, et al. (1994) *Nature* **371**, 757
- [74] Friedmann, A. (1922) On the curvature of Space. *Zeits. f. Physik* **10**, 377.
- [75] Gamow, G., Ivanenko, D. D., Landau, L. D. (1928) World constants and limiting transitions. *J. of Russian Phys. Soc.* **60**, 13-17.
- [76] Gardner, J.P, Cowie, L.L., Wainscoat, R.J., (1993) Galaxy number counts from K=10 to K=23. *Astrophys.J.Lett*, **415**, L9-L12.
- [77] Geller, M. J., Peebles, P. J. (1972) Tests of the expanding Universe postulate. *Astrophys. J.* **174**, 1-5.
- [78] Giovanelli, R., Haynes, M. P. (1986) A 21 cm. survey of the Pisces-Perseus Supercluster. I. The declination zone +27.5 to 33.5 degrees. *Astron. J.* **90**, 24445-73.
- [79] Giovanelli, R., Haynes, M. P. Chincarini, G. L. (1986) Morphological segregation in the Perseus-Pisces Supercluster. *Astrophys. J.*, **300**, 77-92.
- [80] Grischuk, L.P., Petrov, A.N., Popova, A.D. (1984) Exact theory of the (Einstein) gravitational field in an arbitrary background space-time. *Commun. Math. Phys.* **94**, 379-96.
- [81] Guiderdoni, B., Rocca-Volmerange, B. (1990) Constraints on the evolution of high-redshift galaxies and on  $q_0$  from faint galaxy counts. *Astron. Astrophys.*, **227**, 362-378.
- [82] Gush, H.P., Halpern, M., Wisshnow, E.H., (1990) Rocket measurement of the Cosmic-Background-Radiation-mm-wave-spectrum. *Phys.Rev.Lett.* , **65** , 537.
- [83] Guth, A. H. (1981) Inflationary Universe: A possible solution to the horizon and flatness problems. *Phys. Rev. D* **23**, 347-356.
- [84] Guth, A. H.(1992) The big bang and cosmic inflation. *CTP Preprint* **2144**, 1-47.
- [85] Guzzo, L., Iovino, A., Chincarini, G., Giovanelli, R., Haynes, M. P. (1992) Scale-invariant clustering in the large-scale distribution of galaxies. *Astrophys. J. Lett.* **382**, L5-9.
- [86] Haggerty, M. J. (1971) A newtonian model for the creation of a hierarchical cosmology. *Astrophys.J.*, **166**, 257
- [87] Haggerty, M. J., Wertz, J. R. (1972) On the redshift-magnitude relation in hierarchical cosmologies. *Mon. Nat. R. astron. Soc.* **155**, 495-503.
- [88] Harrison, E. R. (1981) Cosmology. Univ. Press, Cambridge.
- [89] Harrison, E. R. (1993) The redshift-distance and velocity-distance laws. *Astrophys. J.* **403**, 28-31.
- [90] Harrison, E. R.(1995) *Astrophys. J.*, 11 June, in press
- [91] Haynes, M., Giovanelli, R., (1988) Large-Scale structure in the Local Universe: the Pisces-Perseus Supercluster, in *Large-scale motion in the Universe*, ed. Rubin, V.C., Coyne, G. , Princeton University Press, Princeton
- [92] Hoyle, F. (1948) A new model for the expanding Universe. *Mon. Nat. R. astr. Soc.* **108**, 372-382.
- [93] Hoyle, F. (1959) *Proc.IAU Symp. 74*, Radio Astronomy & Cosmology, Reidel, Dordrecht, p. 119.

- [94] Hoyle, F., Narlikar, J. V. (1963) Time-symmetric electrodynamics and the arrow of time in cosmology. *Proc. Roy. Soc. London, series A* **277**, 1.
- [95] Hoyle, F., Narlikar, J. V. (1974) Action at distance in physics and cosmology. W. H. Freeman, San Francisco.
- [96] Hoyle, F. (1982) The Universe: past and present reflections. *Ann. Rev. Astron. Astrophys.* **20**, 1-35.
- [97] Hoyle, F. (1991) An assessment of the steady-state theory. In *Modern Cosmology in Retrospect*, Ed. Bertotti, S., Balbinot, R., Bergia, S., Messina, A., pp.221-231, Cambridge Univ. Press.
- [98] Hoyle, F., Burbidge, G., Narlikar, J. V. (1993) A quasi-steady state cosmological model with creation of matter. *Astrophys. J.* **410**, 437-457.
- [99] Hoyle, F., Burbidge, G., Narlikar, J.V., (1994a) Astrophysical deductions from the quasi-steady-state cosmology. *Mon. Not. R. astr. Soc.*, **267**, 1007-1019
- [100] Hoyle, F. , Burbidge, G., Narlikar, J.V., (1994b) Further astrophysical quantities expected in a quasi-steady state Universe. *Astron.Astrophys.* **289**, 729.
- [101] Huchra, J., Davis, M., Latham, D., Tonry, J. (1983) A survey of galaxy redshifts. IV. The data. *Astrophys.J.Suppl.* **52**, 89.
- [102] Hubble, E., (1929) Distance and radial velocity among extra-galactic nebulae. *Nat. Acad. Sci.Proc.* **15**, 168-173.
- [103] Hubble, E., Tolman R., C., (1935) Two methods of investigating the nature of the nebular red-shift. *Astrophys. J.* **82**, 302-337.
- [104] Ivanenko, D. D., Sokolov, A., (1947) Quantum gravitation theory. *Trans. Moskov University.*, N 8, 103-11.
- [105] Jaakkola, T., Moles, M., Vigier, J. P. (1979) Empirical Status in Cosmology and the Problem of the Nature of the Redshifts. *Astron. Nachr.* **300**, 229.
- [106] Jenkins, C. R., Reid, I. N. (1991) A large area determination of galaxy counts at  $2.2\mu\text{m}$ , *Astron. J.* **101**, 1595-600.
- [107] Jones, B. J. T., Martinez, V. J., Saar, E., Einasto, J. (1988) Multifractal description of the large-scale structure of the universe. *Astrophys. J. Lett.* **332**, L1-L5.
- [108] Kaiser, N. (1984) On the spatial correlations of Abell clusters. *Astrophys. J. Lett.* **284**, L9-L12.
- [109] Kalman, G. (1961) Lagrangian formalism in relativistic dynamics. *Phys.Rev.* **123**, 384
- [110] Kapahi, V.K. (1987) *IAU Symp N124 Observational Cosmology*, D.Riedel, 251
- [111] Kellerman (1993) The cosmological deceleration parameter estimated from the angular-size/ redshift relation for compact radio sources. *Nature*, **361**, 134
- [112] Kirshner, R.P., Oelmer, Jr., Schechter, P.L. (1978) A study of field galaxies. I redshifts and photometry of a complete sample of galaxies. *Astron.J.*, **83**,1549
- [113] Landau, L., Lifshiz, E. (1973) Field Theory, Nauka, Moscow
- [114] Lauer, T.R., Postman, M. (1994) The motion of the local group with to the 15,000 kilometer per second Abell cluster inertial frame. *Astrophys.J.* **425**, 418
- [115] LaViolette, P. A. (1986) Is the universe really expanding. *Astrophys. J.* **301**, 544-553.
- [116] Lebedev, V. S., Lebedeva, I. A. (1988) The spatial correlation functions of superclusters of galaxies. *Pis'ma v Astronomicheskii J.* **14**, 1822.
- [117] Lee (1993) Evidence for an old Galactic bulge from RR Lyrae stars in Baade's Window: implications for the formation of the Galaxy and the age of the Universe. *Mon. Not. R. astr. Soc.* **104**, 1780-9.

- [118] Lemson, G., Sanders, R. (1991) On the use of the conditional density as a description of galaxy clustering. *Mon.Not.R.astr.soc.* **252**, 219-28.
- [119] Lilly, S.J., Cowie, L.L., Gardner, J.P. (1991) A deep imaging and spectroscopic survey of faint galaxies. *Astrophys.J.* **369**, 79
- [120] Lilly (1993) A deep-I-band selected galaxy sample: implications for galaxy evolution. *Astrophys.J.*, **411**, 501
- [121] Linde, A. (1987) Particle physics and inflationary cosmology, *Physics Today* , **40**, 61
- [122] Linde, A. (1990) Inflation and Quantum cosmology, Academic Press.
- [123] Linde, A., Linde, D., Mezhlumian, A. (1994) From the Big-Bang theory to the theory of a Stationary Universe. *Phys.Rev.D*, **49**, 1783
- [124] Linde, A. (1994) The self-reproducing inflationary Universe, *Scientific American* , **271**, 32
- [125] Logunov, A. A. Mestvirishvili, M.A., (1989) Relativist Theory of Gravitation. Nauka, Moskov.
- [126] Lynden-Bell, D., Faber, S.M., Burstein, D. , Davies, R.L., Dressler, A., Terlevich, R.J. (1988) Spectroscopy and photometry of elliptical galaxies. V. Galaxy streaming toward the new supergalactic center. *Astrophys.J.* **326**, 19
- [127] Mandelbrot, B. B. (1967) How long is the coast of Britain? Statistical self-similarity and fractal dimension. *Science* **155**, 636-638.
- [128] Mandelbrot, B. B. (1977) Fractals: Form, Chance and Dimension. Ed. by W. H. Freeman, New York.
- [129] Mandelbrot, B. B., (1982) The fractal geometry of nature. Ed. by W. H. Freeman, New York.
- [130] Mandelbrot, B. B.(1988) Fractals and Multifractals: Noise, Turbulence and Galaxies. Springer, New York.
- [131] Martinez, V. T., Jones, B. J. T.(1990) Why the Universe is not a fractal. *Mon. Not. R. astr. Soc.* **242**, 517-21.
- [132] Mather, J. C. et al. (1990) A preliminary measurement of the cosmic microwave background spectrum by the Cosmic Background Explorer (COBE) satellite. *Astrophys.J.Lett*, **354**, L37
- [133] Mather, J. C. et al. (1994) Measurement of the CMB spectrum by COBE FIRAS instrument *Astrophys.J.Lett*, **420**, 439-444
- [134] Mathewson, D.S., Ford. V.L., Buchhorn, M. (1992) No back-side infall into the Great Attractor. *Astrophys.J.Lett.*, **389**, L5
- [135] Mathewson, D.S., Ford. V.L., Buchhorn, M. (1992) A southern sky survey of the peculiar velocities of 1355 spiral galaxies. *Astrophys.J.Suppl. Ser.* **81**, 413-659.
- [136] Mathewson, D.S., Ford, V.L. (1994) Large-scale streaming motions in the local Universe. *Astrophys.J.Lett.* **434**, L39-L42
- [137] Mc Crea, W., Milne, E. (1934) Newtonian universes and the curvature of space. *Quart. J. Mathem.* **5**, 73-80.
- [138] McGaugh, S. (1994) A possible local counterpart to the excess population of faint galaxies. *Nature*, **367**, 538.
- [139] Melchiorri, F., Melchiorri, B. (1994) The Cosmic Background Radiation. *Rivista del nuovo cimento*, **17**, 1.
- [140] Metcalfe, N., Shanks, T., Fong, R., Jones, L.R. (1991) Galaxy number counts. II. CCD observations to B = 25 mag. *Mon. Not. R. astr. Soc.* **249**, 498.
- [141] Meyer, D.M., Black, J.H., Chaffee, F.H., Foltz, C.B., York, D., (1986) An upper limit on the microwave background temperature at  $z = 1.776$ . *Astrophys.J.Lett.* **308**, L37-L41
- [142] Milne, E. A. (1934) A Newtonian expanding Universe. *Quart. J. Mathem.* **5**, 64-72.

- [143] Misner, C., Thorne, K., Wheeler, J. (1973) Gravitation, Freeman, San Francisco
- [144] Mobasher, B., Ellis, R.S., Sharples, R.M. (1986) A complete galaxy redshift survey. *Mon.Not.R.astr.Soc.* **223**, 11.
- [145] Moore, B., Frenk, C.S., Efstathiou, G., Saunders, W. (1994) The clustering of IRAS galaxies. *Mon Not.R.astr.Soc.*, **269**, 742
- [146] Mould J.R. et al. (1991) The velocity field of clusters of galaxies within 100 Mpc. I. Southern clusters. *Astrophys.J.*, **383**, 467
- [147] Murdoch (1977) Recession velocities greater than light, *Quart.J.R.astr.Soc*, **18**, 242-247.
- [148] Narlikar, J. V. (1977) The structure of the Universe. Oxford Univ. Press, London.
- [149] Narlikar, J. V.(1987) Alternative cosmologies in observational cosmology. Ed. Hewitt A., Burbidge G. and Fang L. Z., D. Reidel, Dordrecht, 447-459.
- [150] Narlikar, J. V. (1989) Noncosmological Redshifts. *Space Science Rew.* **50**, 523 -614.
- [151] Narlikar, J. V. (1993) Introduction to Cosmology. Cambr. Univ. Press, London.
- [152] Narlikar, J. V., Arp, H. (1993) Flat spacetime cosmology: a unified framework for extragalactic redshifts. *Astrophys. J.*, **405**, 51-56.
- [153] Nilsson, K. et al. (1993) On the redshift-apparent size diagram of double radio sources *Astrophys. J.*, **413**, 453-476.
- [154] North, J. D. (1965) The measure of the Universe. Claredon Press, Oxford.
- [155] Novikov, I. D.(1988) How did the Universe explode. Nauka, Moscow.
- [156] Ogievetskij, V.I., Polubarinov, I.V. (1965) Interacting Field of Spin 2 and the Einstein Equations. *Ann. Phys.* **35**, 167-208.
- [157] Padovani, P., Urry, C., M. (1992) Luminosity functions, relativistic beaming and unified theories of high-luminosity radio sources. *Astrophys. J.* **387**, 449.
- [158] Pagel, B. E. J., Simonson, E. A., Terlevich, R. J., Edmunds, M. G. (1992) The primordial helium abundance from observations of extragalactic H II regions. *Mon. Not. R. astr. Soc.* **255**, 325-345.
- [159] Paladin, G., & Vulpiani, A. (1987) Anomalous scaling laws in multifractal objects *Phys. Rep.*, **156**, 147.
- [160] Pantyushin, A.A. (1972) Theory of direct gravitational interaction. *PhD thesis* , Alma-Ata.
- [161] Park, C., Vogeley, M.S., Geller, M., Huchra, J. (1994) Power spectrum, correlation, and tests for luminosity bias in the CfA redshift survey. *Astrophys.J.* **431**, 569
- [162] Paturel, G., Bottinelli, L., Gouguenheim, L. Fouque, P. (1988) New determination of the pole of a "hypergalactic" large-scale system *Astronom. Astrophys.* **189**, 1
- [163] Paturel, G., Di Nella, H., Bottinelli, L., Fouque, P., Gouguenheim, L. (1994) Interpretation of a complete curve *Astronom. Astrophys.* in press.
- [164] Peebles, P. J. E. (1980) The Large-Scale Structure of the Universe. Princeton Univ. Press, Princeton, New Jersey.
- [165] Peebles, P. J. E. (1988) The theoretical aspects of the nebular redshift, 33 years later. *Pubbl. Astron. Soc. pacif.* **100**, 670-679.
- [166] Peebles, P.J.E. (1988) The local extragalactic velocity field as a test of the explosion and gravitational instability pictures. *Astrophys.J.*, **332**, 17



- [167] Peebles, P. J. E., Schramm, D. N., Turner, E. L., Kron, R. G. (1991) The case for the relativistic hot Big Bang cosmology. *Nature* **352**, 769-776.
- [168] Peebles, P.J.E. (1993) Principles of physical cosmology, Princeton Univ. Press. Princeton
- [169] Peebles, P. J. E., Schramm, D., Turner, E.L., Kron, R.G. (1994) The evolution of the Universe, *Scientific American*, **316**, 26
- [170] Pence, W. (1976) K-corrections for galaxies of different morphological types. *Astrophys.J.* **203**, 39-51.
- [171] Penzias, A.A., Wilson, R.W. (1965) A measurement of excess antenna temperature at 4080 Mc/s. *Astrophys.J.*, **142**, 419
- [172] Perdan, J. (1990) Astrophysical fractals: an overview and prospects. *Vistas in Astron.* **33**, 249-294.
- [173] Pierce, M.J., Welch, D.L., McClure, R.D., van der Bergh, S., Raicine, R., Stetson, P.B. (1994) The Hubble constant and Virgo cluster distance from observations of Cepheid variables. *Nature* , **371**, 385
- [174] Pietronero, L., Tosatti, E., eds. (1986) Fractal in Physics. North-Holland, Amsterdam.
- [175] Pietronero, L. (1987) The fractal structure of the Universe: correlations of galaxies and clusters and the average mass density. *Physica A.*, **144**, 257
- [176] Pietronero, L., Sylos Labini, F. (1994) Cosmological Principle and the debate about large-scale Structure distribution. in *Birth of the Universe and fundamental physics*, F. Occhionero ed., Springer Verlag, in press
- [177] Pietronero, L., Sylos Labini, F. (1995) Comments on the CfA 2 analysis, *preprint*
- [178] Press, W. H., Schechter, P. (1974) Formation of galaxies and clusters of galaxies by self-similar gravitational condensation. *Astrophys. J.* **187**, 425-38.
- [179] Provenzale, A., Guzzo, L., Murante, G. (1994) Clustering properties from finite galaxy samples. *Mon. Not. R. astr.Soc.* **226**, 555.
- [180] Refsdal, S., Surdej J. (1994) Gravitational lenses. *Rep. Prog. Phys.***56**, 117-185.
- [181] Ribeiro (1992a) On modeling a relativistic hierarchical (fractal) cosmology by Tolman's spacetime: II analysis of the Einstein- de Sitter model. *Astrophys.J.*, **395**, 29
- [182] Ribeiro (1992b) On modeling a relativistic hierarchical (fractal) cosmology by Tolman's spacetime: I theory. *Astrophys.J.*, **388**, 1
- [183] Ribeiro (1993) On modeling a relativistic hierarchical (fractal) cosmology by Tolman's spacetime: III Numerical results. *Astrophys.J.*, **415**, 469
- [184] Rubin, V. C., Thonnard, N., Ford, W.K., Roberts, M.S. (1976) Motion of the galaxy and the Local Group determined from the velocity anisotropy of distant Sc I galaxies. II. The analysis for the motion. *Astron.J.*, **81**, 719
- [185] Sachs, R.K., Wolf, A.M. (1967) Perturbations of a cosmological model and angular variations of the microwave background. *Astrophys.J.*, **147**, 73.
- [186] Sandage, A. (1986) The redshift-distance relation IX. Perturbation of the very nearby velocity fields by the mass of Local Group *Astrophys. J.* **307**, 1-19.
- [187] Sandage, A. (1987) Observational Cosmology 1920-1985: an introduction to the conference, in *Observational Cosmology*. Ed. Hewitt A., Burbidge G. and Fang L. Z., 1-27 D. Reidel, Dordrecht.
- [188] Sandage, A. (1988) Observational tests of world models. *Ann. Rev. Astron. Astrophys.* **26**, 561-630.
- [189] Sandage, A., Tamman, G.A., (1994) Two lectures on the observational determination of the Hubble expansion rate, OCIW preprint pg. 1-19.

- [190] Sandage, A., Perelmuter, J. M. (1991) The surface brightness test for the expansion of the Universe. III. Reduction of data for the several brightest galaxies in clusters to standard conditions and a first indication that the expansion is real. *Astrophys. J.* **370**, 455-73.
- [191] Saunders, W., Frenk, C. Rowan-Robinson, M., Efstathiou, G., Lawrence, A., Kaiser, N., Ellis, R., Crawford, J., Xia, X., Parry, I. (1991) The density of the local Universe. *Nature* **349**, 32-38.
- [192] Shanks, T., Stevenson, P.R.F., Fong, R. MacGillivray, H.T. (1984) Galaxy number counts and cosmology. *Mon.Not.R.astr.Soc.*, **206**, 767
- [193] Schechter, P. (1976) An analytic expression of the luminosity function of galaxies. *Astrophys.J.*, **203**, 297
- [194] Schmidt, B. P. et al.(1994) The distance of Five Type II Supernovae using the expanding photosphere method and the value of  $H_0$ . *Astrophys.J.*, **434**, L19, 232-D2.
- [195] Schneider, P., Ehlers, J., Falco, E. E. (1992) Gravitational lensing. Springer Verlag, Berlin.
- [196] Schombert, J. M., Astron, J. (1988) A catalog of low-surface-brightness objects. I. Declination zone  $+20^\circ$ . *Astrophys. J.*, **95**, 634-60.
- [197] Segal, I. (1986) Directly observed relations in complete galaxy samples and the predictions of redshift-distance power laws. *Proc. Nat. Acad. Scie.* **83**, 7129-31.
- [198] Smoot, G. et al. I. (1992) Structure in the COBE microwave radiometer first-year maps *Astrophys. J.* **396**, L1-L5.
- [199] Sokolov, V. V., Baryshev, Yu. V., (1980) Field theory approach to gravity. Energy momentum tensor of the field. *Gravitation and Relativity Theory, KGU*, **vyp.17** , 34-42.
- [200] Sokolov, V. V. (1992) Linear and nonlinear gravodynamics: static field of a collapsar. *Astrophys.Sp.Sci.***191**, 231-258.
- [201] Songaila, A., Cowie, L. L., Hogan, C. J., Rugers, M. (1994) Deuterium abundance and background radiation temperature in high-redshift primordial clouds. *Nature* **368**, 599-603.
- [202] Songaila, A., et al. (1994) Measurements of the microwave background temperature at redshift of 1.775. *Nature* **371**, 43-45.
- [203] Spinrad H, Dey, A., Graham, J.R. (1995) Keck observations of the most distant galaxies: 8C 1435+63 at  $z=4.25$  *Astrophys. J.*, **438**, L54-L57
- [204] Stecker, F. W. (1982) A new view of baryon symmetric cosmology based on a grand unified theories. In *Progress in Cosmology*, Proceedings of the Oxford International Symposium (Dordrecht, Reidel 1982).
- [205] Steigman, G., Tosi, M. (1992) Galactic evolution of D and  $^3\text{He}$ . *Astrophys. J.* **401**, 150-156.
- [206] Strukov et al., (1992) Anisotropy of the microwave background radiation *Sov. Astron. Lett* **18**, 153-156.
- [207] Sylos Labini, F., (1994) Isotropy, Homogeneity and dipole saturation. *Astrophys.J.*, **433**, 464-67.
- [208] Sylos Labini, F., Pietronero, L. (1994a) Stochastic aggregation model for the multifractal distribution of matter, in *Birth of the Universe and fundamental physics*, F. Occhionero ed., Springer Verlag, in press
- [209] Sylos Labini, F., Pietronero, L. (1994b) Multifractal properties of visible matter distribution: a stochastic model. Proceedings of *III Congresso Nazionale di Cosmologia*, in press
- [210] Sylos Labini, F., Amendola, L., Pietronero, L. (1995) Power spectrum of self-similar distribution. *in preparation*
- [211] Sylos Labini, F., Pietronero, L. (1995a) Number redshift relation in the ESP survey. *in preparation*
- [212] Sylos Labini, F., Pietronero, L., et al. (1995b) Multifractal properties of visible matter distribution. *in preparation*

- [213] Sylos Labini, F., Pietronero, L., (1995c) Number counts and spatial correlation properties. *in preparation*
- [214] Symbalisky, E.M., Schramm, D.N., (1981) Nucleocosmochronology. *Rep.Prog.Phys.*, **44**, 293
- [215] Taylor J.H. et al., (1992) Experimental constraints on strong-field relativistic gravity *Nature***355**, 132-136
- [216] Teerikorpi, (1987) Cluster population incompleteness bias and distances from the Tully-Fisher relation: theory and numerical examples. *Astron.Astrophys.* , **173**, 39
- [217] Teerikorpi, P., Bottinelli, L., Gouguenheim, L., Paturel, G. (1992) Investigation of the local supercluster velocity field I. *Astron.Astrophys.* , **260**, 17
- [218] Teerikorpi, P. (1993) On general Malmquist corrections to direct and inverse Tully-Fisher distance moduli. *Astron.Astrophys.* , **280**, 443-450.
- [219] Thirring, W. E., (1961) An alternative approach to the theory of gravitation. *Ann. of Phys.* **16**, 96-117.
- [220] Thuan, T. X., Gott III, J. R., Schneider, S. E. (1987) The spatial distribution of dwarf galaxies in the *CfA* slice of the Universe. *Astrophys. J. Lett.* **315**, L93-7.
- [221] Tolman, R.C., (1934) Effect of inhomogeneity in cosmological models. *Proc. N.A.S.*, **20**, 169.
- [222] Troitskii, V. S.(1987) Physical constants and the evolution of the Universe. *Ap.Sp.Sci* **139**, 389-411
- [223] Tully, R.B., (1986) Alignment of clusters and galaxies on scales up to 0.1c *Astrophys. J.*, **303**, 25
- [224] Tully, R.B., (1987) More about clustering on a scale of 0.1c *Astrophys. J.*, **323**, 1
- [225] Tully, R.B., Fisher, J.R. (1977) A new method of determining distances to galaxies. *Astronom. Astrophys.*, **54**, 661
- [226] Tully, R. B., Scaramella, R., Vettolani, G., Zamorani, G. (1992) Possible geometric patterns in 0.1c scale structure. *Astrophys. J.* **388**, 9-16.
- [227] Turner, E.L., Ostriker, J.P., Gott, J.R. (1984) The statistics of gravitational lenses: the distributions of image angular separations and lens redshifts. *Astrophys.J.*, **284**, 1
- [228] Tyson, J. A. (1988) Deep CCD survey: galaxy luminosity and color evolution. *Astronom.J.*, **96**, 1
- [229] Van der Berg (1983) Globular clusters and galaxy formation. *Comm. Astrophys. Comm. Mod. Phys. Part C* **10**, 27.
- [230] Van den Bergh (1990) What are S0 galaxies ? *Astrophys.J.*, **348**, 57
- [231] Vettolani, G., et al. (1994) Preliminary results from the ESO slice project (ESP). Proc. of Schloss Rindberg workshop. *Studying the Universe with Clusters of Galaxies*.
- [232] Vigier, J.P. (1988) Alternative interpretation of the cosmological redshift in terms of vacuum gravitational drag, in *New Ideas in Astronomy*, Cambridge, 257-274.
- [233] Von Hoerner, S. (1974) Cosmology in Galactic and Extragalactic Radio Astronomy. Springer-Verlag.
- [234] Wheeler, J. A. and Feynman, R. P. (1949) Classical Electrodynamics in Terms of Direct Interparticle action. *Rev. Mod. Phys.* **21**, 425-433.
- [235] Whitrow, G.J., Morduch, G.E. (1965) Relativistic theories of gravitation. *Vistas in Astronomy*, **6**, 1-67.
- [236] Weinberg, S. E. (1965) Photons and gravitons in perturbation theory: derivation of Maxwell's and Einstein's equations. *Phys.Rev.* **138B**, 988-1002.
- [237] Weinberg, S. E. (1972) Gravitation and Cosmology. Wiley, New York.
- [238] Weinberg, S. E. (1989), The cosmological constant problem. *Rev.Mod..Phys.*, **61**,1.

- [239] Wertz, J. R. (1970) Newtonian hierarchical cosmology. *Univ. Texas Publ. Astron.* **N3**, 3-143.
- [240] Willick, (1990) The peculiar velocity field in the Perseus-Pisces supercluster. *Astrophys.J.Lett.*, **351**, L5-L8.
- [241] Yoshii, Yu., Takahara, F. (1988) Galactic evolution and cosmology: probing the cosmological deceleration parameter. *Astrophys. J.* **326**, 1-18
- [242] Yoshii, Yu. (1993) Detection and selection effects in observations of faint galaxies. *Astrophys. J.* **403**, 552-566.
- [243] Zwicky, F. (1929) On the redshift of spectral lines through interstellar space. *Proc. N.A.S.*, **15**, 773

# FACTS AND IDEAS IN MODERN COSMOLOGY

## 1. INTRODUCTION

## 2. GENERAL BASIS OF COSMOLOGY

### 2.1 Empirical basis for cosmological theories

### 2.2. A classification of cosmological theories

#### 2.2.1. Gravitation theories

#### 2.2.2. Matter distribution

#### 2.2.3. The nature of the redshift

#### 2.2.4. CMBR nature

#### 2.2.5. Evolution and the arrow of time

#### 2.2.6. Main rival cosmological theories

### 2.3. Classification of cosmological tests

### 2.4. Biases and selection effects in astronomical data

## 3. THEORETICAL PREDICTIONS OF COSMOLOGICAL MODELS

### 3.1. Paradoxes of Newtonian cosmology and origin of modern cosmological ideas

### 3.2. Big Bang models

#### 3.2.1. Initial hypotheses of the Standard Friedmann Model

#### 3.2.2. Successes of the Standard Model

#### 3.2.3. Crucial tests

#### 3.2.4. Parametric tests

##### *3.2.4.1. The proper distance-redshift relation*

##### *3.2.4.2. The angular size-redshift relation*

##### *3.2.4.3. The magnitude-redshift relation*

##### *3.2.4.4. The time-redshift relation*

##### *3.2.4.5. Count-redshift and count-magnitude relations*

#### 3.2.5. Paradoxes of the standard model

#### 3.2.6. Non-standard models

##### *3.2.6.1. $\Lambda$ -term*

##### *3.2.6.2. Inflationary Universe*

### 3.3. Steady-State Models

#### 3.3.1. Basic hypothesis of the SSM

#### 3.3.2. Predictions for testing

#### 3.3.3. Modern developments of the SSM

### 3.4. Fractal Models

#### 3.4.1. Fractal geometry

#### 3.4.2. Fractal structures

*3.4.2.1. Mathematical Self-similarity*

*3.4.2.2. "Linear and non-linear dynamics"*

*3.4.2.3. Fair sample*

*3.4.2.4. Isotropy, homogeneity and Cosmological Principle*

*3.4.2.5. Power Spectrum*

#### 3.4.3. Fractal Properties of visible matter

*3.4.3.1. Galaxies and clusters*

*3.4.3.2. The CfA2 redshift survey*

*3.4.3.3. Pencil beams and very deep surveys*

#### 3.4.4. Multifractal Properties of luminous matter distribution

*3.4.4.1. Luminosity function*

*3.4.4.2. Number counts*

*3.4.4.3. Magnitude-redshift relation*

*3.4.4.4. Luminosity segregation*

*3.4.4.5. A simple stochastic model for the formation of a MF distribution*

*3.4.4.6. Gravitational lensing in a fractal distribution*

#### 3.4.5. Newtonian fractal models

#### 3.4.6. General relativity Fractal models

#### 3.4.7. Field Fractal models

### 3.5. Other possibilities

## 4. CONFRONTATION OF WORLD MODELS WITH OBSERVATIONS

### 4.1 Parametric cosmological tests

#### 4.1.1. Redshift-distance relation and the Hubble constant

#### 4.1.2. Angular size-redshift relation

#### 4.1.3. Apparent magnitude-redshift relation

#### 4.1.4. Galaxy counts

#### 4.1.5. Other parametric tests

*4.1.5.1. Ages of objects*

*4.1.5.2. Galaxy bulk flows*

*4.1.5.3. Gravitational lensing*

*4.1.5.4. Light element abundances*

### 4.2 Crucial cosmological tests

#### 4.2.1. Experimental testing of the relativistic gravity

#### 4.2.2. Redshift nature

#### 4.2.3. Large-scale matter distribution

#### 4.2.4. Cosmic Microwave Background Radiation

## 5. DISCUSSION AND CONCLUSIONS

## 6. REFERENCES